20MA4T1- STATISTICS AND NUMERICAL METHODS

FOR II - B.E. (MECHANICAL) / IV SEMESTER

SYLLABUS

Semester	Programme	Course Code	Course Name	L	T	P	C
IV	B.E. MECH	20MA4T1	STATISTICS AND NUMERICALMETHODS	3	1	0	4

COURSE LEARNING OUTCOMES (COs)			
After Successful completion of the course, the students should be able to		RBT Level	Topics Covered
CO1	Identify and apply various numerical techniques for solving non-linear equations and systems of linear equations.	K3	3
CO2	Analyse and apply the knowledge of interpolation and determine the integration and differentiation of the functions by using the numerical data.	K4	4
CO3	Justify the concept of testing of hypothesis for small and large samples and interpret the results.	K5	1
CO4	Classify the principles of design of experiments and perform analysis of variance.	K2	2
CO5	Determine the dynamic behaviour of the system through solution of ordinary differential equations by using numerical methods.	K5	5

PRE-REQUISITE	Engineering Mathematics I , Engineering Mathematics II and Transforms and Partial Differential Equations

CO / PO MAPPING (1 – Weak, 2 – Medium, 3 – Strong)														
COs	Programme Learning Outcomes (POs)												PSOs	
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1	3	3		3				1	3	3		3		
CO2	3	3		3				1	3	3		3		
CO3	3	3		3				1	3	3		3		
CO4	3	3		3				1	3	3		3		
CO5	3	3		3				1	3	3		3		

COURSE ASSESSMENT METHODS		
DIRECT	1	Continuous Assessment Tests
	2	Assignments and tutorials
	3	End Semester Examinations
INDIRECT	1	Course Exit Survey

COURSE CONTENT		
Topic - 1	**TESTING OF HYPOTHESIS**	**9 + 3**
Sampling distributions – Estimation of parameters – Statistical hypothesis – Large sample tests based on Normal distribution for single mean and difference of means -Tests based on t, Chi-square and F distributions for mean, variance and proportion – Contingency table (test for independent) – Goodness of fit.		
Topic - 2	**DESIGN OF EXPERIMENTS**	**9 + 3**
One way and two way classifications – Completely randomized design – Randomized block design – Latin square design – 2^2 factorial design.		
Topic - 3	**SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS**	**9 + 3**
Solution of algebraic and transcendental equations – Fixed point iteration method – Newton Raphson method – Solution of linear system of equations – Gauss elimination method – Pivoting – Gauss Jordan method – Iterative methods of Gauss Jacobi and Gauss Seidel – Eigen values of a matrix by Power method.		
Topic - 4	**INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION**	**9 + 3**
Lagrange's and Newton's divided difference interpolations – Newton's forward and backward difference interpolation – Approximation of derivates using interpolation polynomials – Numerical single and double integrations using Trapezoidal and Simpson's 1/3 rules.		
Topic - 5	**NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS**	**9 + 3**
Single step methods : Taylor's series method – Euler's method – Modified Euler's method – Fourth order Runge-Kutta method for solving first order equations – Multi step methods : Milne's predictor corrector methods for solving first order equations.		

THEORY	**45**		**TUTORIAL**	**15**		**PRACTICAL**	**0**		**TOTAL**	**60**

BOOK REFERENCES	
1	Gerald. C. F. and Wheatley. P. O., **"Applied Numerical Analysis"**, Pearson Education,Asia, 7th Edition, New Delhi, 2006.
2	Grewal, B.S., and Grewal, J.S., **"Numerical Methods in Engineering and Science"**, Khanna Publishers, 9th Edition, New Delhi, 2010
3	Burden, R.L and Faires, J.D, **"Numerical Analysis",** 9th Edition, Cengage Learning, 2016.

4	Vijay K. Rohatgi , EhsanesSaleh,**"An Introduction to Probability and Statisics",** 2nd Edition,2009
5	N. G. Das**.,"Statistical Methods"**, Tata McGraw Hill Publishing Ltd,2008

OTHER REFERENCES	
1	**https://www.sobtell.com/blog/38-real-life-applications-of-numerical-analysis**
2	**https://www.scienceabc.com/eyeopeners/why-do-we-need-numerical-analysis-in-everyday-life.html**
3	**https://leverageedu.com/blog/application-of-statistics/**

UNIT-I

TESTING OF HYPOTHESIS

PART – A

TESTING OF HYPOTHESIS

1. **Explain the terms sample size and sampling error**
   Solution:
   Sample size: A finite subset of statistical individuals in a population is called a sample and the number of individuals in a sample is called sample size.
   Sampling error: For the purpose of determining population characteristics, the individuals in the sample are observed. On examining the sample of a particular stuff we arrive at a decision of purchasing or rejecting that stuff. The error involved in such approximation is known as sampling error.
2. **Define the following terms: Statistics, Parameter and Standard Error**
   Solution:
   Statistic: Measure describing the characteristic of sample
   Parameter: Values that describe the characteristic of population
   Standard error: Standard deviation of the sampling distribution of a statistic

3. **Define Type I and Type II errors in taking a decision.**
   Solution:

   Type I Error: The hypothesis is true but our tests reject it.

   Type II Error: The hypothesis is false but our test accepts it.

	Decision	
	Accept H_0	Reject H_0
H_0 True	Correct decision	Type I error
H_0 False	Type II error	Correct decision

4. **State the proceedure involved in testing of hypothesis.**
   Solution:
   (i) Set up a null hypothesis H_0,
   (ii) Set up the alternative hypothesis H_1,
   (iii) Select the appropriate level of significance (α),
   (iv) Compute the test statistic $z = \frac{t-E(t)}{sE(t)}$ under H_0,
   (v) We compare the "calculate z" with "critical value z_α" at given level of significance (α)

   [If$|z| < 1.96$, H_0 may be accepted at 5% level of significance.

   If $|z| > 1.96$, H_0 may be rejected at 5% level of significance.

   If $|z| > 2.58$, H_0 may be accepted at 1% level of significance.

   If $|z| > 2.58$, H_0 may be rejected at 1% level of significance.

5. **What is meant by level of significance and critical region?**

Solution:

Critical region:

A region corresponding to a statistics t in the sample space S which lead to the rejection of H_0 is called c**ritical Region** or **Rejection Region.**Those region which lead to the acceptance of H_0 give as a region alled**Acceptance Region.**

Level of significance:

The probability α' that a random value of the statistic 't' belongs to the critical region is known as the level of significance. In other words, level of significance is the size of Type I Error. The level of significance usually employed in testing of hypothesis are 5% and 1%.

6. **Define Null and Alternative hypothesis.**

Solution:

Null hypothesis is based for analyzing the problem Null hypothesis is the hypothesis of no difference and is denote byH_0.

Any hypothesis which is complementary to the null hypothesis is called an Alternative hypothesis, denoted by H_1

7. **A coin is tossed 400 times and it turns up head 216 times. Discuss whether the coin may be unbiased one at 5% level of significance.**

Solution:

Given $n = 400, P = \frac{1}{2}$

$Q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

X=Number of success=216

(i) The parameter of interest is P.

(ii) H_0: The coin is unbiased

(iii) H_1: The coin is biased

(iv) $\alpha = 0.05$

(v) $Z = \frac{X-np}{\sqrt{npQ}}$

(vi) Reject H_0 if $|z| > 1.96$

(vii) Computation:$Z = \frac{216-(400)\left(\frac{1}{2}\right)}{\sqrt{(400)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}} = \frac{16}{10}$

(viii) Conclusion: $|Z| = 1.6 < 1.96$

So we accept H_0 at 5% level of significance.

Hence coin is unbiased.

8. A standard sample of 200 tins of coconut oil gave an average weight of 4.95Kgs. With a standard deviation of 0.21 Kg.Do we accept that the net weight is 5 Kgs per tin at 5% level of significance?

Solution:

Given $n = 200, \bar{x} = 4.95\ kg, \sigma = 0.21\ kg,\ \mu_0 = 5\ kg$

(i) The parameter of interest is μ.

(ii) $H_0: \mu = 5\ kgs$ [The net weight is 5 kgs]

(iii) $H_1: \mu \neq 5\ kgs$ [Two tailed test]

(iv) $\alpha = 0.05$

(v) $Z = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$

(vi) Reject H_0 if $|z| > 1.96$ at $\alpha = 0.05$

(vii) Computation: $Z = \frac{4.95-5}{\left(\frac{0.21}{\sqrt{200}}\right)} = -3.36$

(viii) Conclusion: $|Z| = 3.36 > 1.96$

So we reject $H_0: \mu = 5\ kgs$ at 5% level of significance.

t-DISTRIBUTION

9. Write down the formula of test statistic t to test the significance of difference between the means of large samples.

Solution:

$$Z = \frac{|\overline{x_1} - \overline{x_2}|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

CHI-SQUARE (χ^2) DISTRIBUTION

10. Define Chi-Square test for goodness of fit.

Solution:

Chi-Square test for goodness of fitis a test to find if the deviation of the experiment from theory is just by chance or it is due to the inadequacy of the theory to fit the observed data. By this test, we test whether difference between observed and expected frequencies are significant or not.

Chi-Square test for goodness of fit is defined by

$\chi^2 = \sum \frac{(O-E)^2}{E}$. Where O-Observed frequency

E- Expected frequency

11. Write any two applications of Chi − square (χ^2) – test.

Solution:

(i) To test the goodness of fit

(ii) To test the "independence of attributes"

(iii) To test the homogeneous of independent estimations.

12. What are the conditions for the validity of Ψ^2- test? OR State the conditions for applying Ψ^2- test.

Solution:

(i) The sample observations should be independent.
(ii) Constraints on the cell frequencies, if any, must be linear [e.g., $\sum O_i = \sum E_i$]
(iii) N, the total frequency, should be at least 50.
(iv) No theoretical cell frequency should be less than 5.

13. What are the expected frequencies of 2*2 contigency table

a	**b**
c	**d**

Solution:

Expected frequency table:

$\frac{(a+b)(a+c)}{N}$	$\frac{(a+b)(b+d)}{N}$
$\frac{(a+c)(c+d)}{N}$	$\frac{(c+d)(b+d)}{N}$

14. Give the formula for the χ^2- test of independence for

a	**b**
c	**d**

Solution:

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}, \qquad N = a+b+c+d$$

PART – B

TESTING OF HYPOTHESIS FOR SINGLEMEAN

1. **Explain clearly the procedure generally followed in testing of a hypothesis.**

Solution:

General procedure for hypothesis tests

i. From the problem context, identify the parameter of interest.

ii. State the null hypothesis, H_0 .

iii. Specify an appropriate alternative hypothesis, H_1.

iv. Choose a significance level α.

If $|z| < 1.96, H_0$ may be accepted at 5% level of significance.

If $|z| > 1.96, H_0$ may be rejected at 5% level of significance.

If $|z| < 2.58, H_0$ may be accepted at 1% level of significance.

If $|z| > 2.58, H_0$ may be rejected at 1% level of significance.

v. Determine an appropriate test statistic.

vi. State the rejection for the statistic.

vii. Compute any necessary sample quantities, substitute these into the equation for the test statistic and compute the value.

viii. Conclusion: Decide whether or not, H_0 should be rejected and report that in the problem context.

2. A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cms and standard deviation of 2.61 cms? (Test at 5% level of significance. The value of z at 5% level is $|Z_{\propto}| < 1.96$).

Solution:

Given $n = 900 \mu_0 = 3.25$

$$\bar{x} = 3.4\ cm \sigma = 2.61$$

$$s = 2.61$$

(i) The parameter is μ_0.

(ii) Null hypothesis H_0: Assume that the sample has been drawn from the population with mean $\mu = 3.25$.

(iii) Alternative hypothesis $H_1: \mu \neq 3.25$

(iv) Level of significance $\propto = 0.05$.

(v) The test statistic is $z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$

(vi) Reject if $|z| > 1.96$ at 5% level of siginificance.

(vii) **Computation:**

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$= 1.724$$

$$z = 1.724 < 1.96$$

(viii) **Conclusion**

Here $|z| = 1.724 < 1.96$, so we accept the null hypothesis H_0 at 5% level of significance.

<u>TESTING OF HYPOTHESIS FOR DIFFERENCE OF MEAN</u>

3. The means of two large samples of 1000 and 2000 members are 67.5 and 68.0 inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches?

Solution:

Given sample sizes $n_1 = 1000,\ n_2 = 2000$

$$\bar{x}_1 = 67.5, \bar{x}_2 = 68.0$$

$$s_1 = \sigma_1 = 2.5 \quad s_2 = \sigma_2 = 2.5$$

(i) The parameter is $\mu_1 \& \mu_2$.

(ii) Null hypothesis H_0: $\mu_1 = \mu_2$ (there is no significant difference).

(iii) Alternative hypothesis $H_1: \mu_1 \neq \mu_2$

(iv) Level of siginificance $\propto = 0.05$.

(v) The test statistic is $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$

(vi) Reject if $|z| > 1.96$ at 5% level of siginificance.

(vii) Computation:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$$

$$= \frac{67.5 - 68.0}{\sqrt{\frac{2.5^2}{1000} + \frac{2.5^2}{2000}}} = \frac{-0.5}{0.09685} = -5.16$$

$$|z| = 5.16$$

(viii) Conclusion

Here $|z| = 5.16 > 1.96$, so we reject the null hypothesis H_0 at 5% level of significance.

4. A random sample of 100 bulbs from a company P shows a mean life 1300 hours and standard deviation of 82 hours. Another random sample of 100 bulbs from company Q showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company P superior to bulbs of company Q at 5% level of significance?

Solution:

$H_0: \mu_1 = \mu_{2,}$

$H_0: \mu_1 > \mu_{2,}$(right tailed test)

L.O.S $\alpha = \frac{5}{100} = 0.05.$

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$

Where $n_1 = 100, \quad n_2 = 100$

$\bar{x}_1 = 1300, \quad \bar{x}_2 = 1248$

$s_1 = \sigma_1 = 82 \quad s_2 = \sigma_2 = 93$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$$

$$= \frac{1300 - 1248}{\sqrt{\frac{82^2}{100} + \frac{93^2}{100}}}$$

$$= \frac{52}{12.39879} = 4.1939$$

Table value of z at 5% LOS is 1.675.

Since 4.19 > 1.645, H_0 is rejected and the bulbs of company A is superior to the bulbs of company B.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}}}$$

$$= \frac{67.5 - 68.0}{\sqrt{\frac{2.5^2}{1000} + \frac{2.5^2}{2000}}} = \frac{-0.5}{0.09685} = -5.16$$

5. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls.

Solution: **(AU N/D 2012, M/ J 2016)**

	No of cases	Mean	S.D
Sample I	**50**	**76**	**6**
Sample II	**75**	**82**	**2**

Given sample sizes $n_1 = 50,$ $n_2 = 75$

$$\bar{x}_1 = 76, \bar{x}_2 = 82$$

$$s_1 = \sigma_1 = 6 \qquad s_2 = \sigma_2 = 2$$

The parameter is $\mu_1 \& \mu_2$.

(i) Null hypothesis H_0: $\mu_1 = \mu_2$ (there is no significant difference).

(ii) Alternative hypothesis H_1: $\mu_1 \neq \mu_2$

(iii) Level of significance $\propto = 0.05$.

(iv) The test statistic is $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}}}$

(v) Reject if $|z| > 1.96$ at 5% level of siginificance.

(vi) Computation:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}} = \frac{76 - 82}{\sqrt{\frac{6^2}{50} + \frac{2^2}{75}}}$$

$$= \frac{-6}{\sqrt{\frac{36}{50} + \frac{4}{75}}} = -\frac{6}{0.88}$$

$$= -6.82$$

$$|z| = 6.82$$

(vii) Conclusion

Here $|z| = 6.82 > 1.96$, so we reject the null hypothesis H_0 at 5% level of significance.

6. Test if the variances are significantly different for

X1	**24**	**27**	**26**	**21**	**25**	
X2	**27**	**30**	**32**	**36**	**28**	**23**

Solution:

x	$x - \bar{x}$	$(x - \bar{x})^2$	Y	$y - \bar{y}$	$(y - \bar{y})^2$
24	-0.6	0.36	27	-2.3	5.29
27	2.4	5.76	30	0.7	0.49
26	1.4	1.96	32	-2.7	7.29
21	-3.6	12.96	36	6.7	44.89
25	0.4	0.16	28	-1.3	1.69
			23	-6.3	39.69
123	**0**	**21.2**	**176**	**-5.2**	**99.34**

$$\bar{x} = \frac{123}{5} = 24.6, \ \bar{y} = \frac{176}{6} = 29.3$$

$$\sum (x - \bar{x})^2 = 21.2, \qquad \sum (y - \bar{y})^2 = 99.34$$

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum (x - \bar{x})^2 + \sum (y - \bar{y})^2\right]$$

$$= \frac{1}{5 + 6 - 2}[21.2 + 99.34] = 13.39$$

$$s = 3.66$$

(i) The parameter is $\mu_1 \& \mu_2$.

(ii) Null hypothesis $H_0 : \mu_1 = \mu_2$ (there is no significant difference).

(iii) Alternative hypothesis $H_1: \mu_1 \neq \mu_2$

(iv) Level of significance $\propto = 0.05$.

(v) The test statistic is $z = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

(vi) Reject if $|z| > 2.262$ at 5% level of significance for 9 degrees of freedom

(vii) Computation:

$$z = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{24.6-29.3}{3.66\sqrt{\frac{1}{5}+\frac{1}{6}}} = \frac{-4.7}{2.2172} = -2.1197$$

$|z| = 2.12$

(viii) Conclusion

Here $|z| = 2.12 < 2.262$, so we accept the null hypothesis H_0 at 5% level of significance

Hence there is no significant difference.

TESTING OF HYPOTHESIS FOR DIFFERENCE OF PROPORTION

t-DISTRIBUTION(n<30)

7. Test if the difference in the means is significant for the following data:

Sample I	**76**	**68**	**70**	**43**	**94**	**68**	**33**	
Sample II	**40**	**48**	**92**	**85**	**70**	**76**	**68**	**22**

Solution:

Calculation for sample means and S.D.'s

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
76	11.4	129.96	40	-22.6	510.76
68	3.4	11.56	48	-14.6	213.16
70	5.4	29.16	92	29.4	864.36
43	-21.6	466.56	85	22.4	501.76
94	29.4	864.36	70	7.4	54.76
68	3.4	11.56	76	13.4	179.56
33	-31.6	998.56	68	5.4	29.16
			22	-40.6	1648.36
452		2511.72	501		4001.88

Given $n_1 = 7,\ n_2 = 8\ (< 30\ so\ we\ use\ t-test)$

Mean $\bar{x} = \frac{452}{7} = 64.6$ $\bar{y} = \frac{501}{8} = 62.6$

$$\sum(x-\bar{x})^2 = 2511.72 \qquad \sum(y-\bar{y})^2 = 4001.88$$

$$s^2 = \frac{\sum(x-\bar{x})^2 + \sum(y-\bar{y})^2}{n_1+n_2-2} = \frac{2511.72 + 4001.88}{7+8-2} = 501.04$$

$$s = 22.38$$

(i) The parameter is $\mu_1 \& \mu_2$.

(ii) Null hypothesis $H_0: \mu_1 = \mu_2$ (there is no significant difference in the variability in yields).

(iii) Alternative hypothesis $H_1: \mu_1 \neq \mu_2$

(iv) Level of significance $\propto = 0.05$. d.f$= 7+8-2 = 13$

(v) The test statistic is $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

(vi) Reject if $|t| > 2.16$ at 5% level of significance.

(vii) Computation:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{64.6 - 62.6}{22.38\sqrt{\frac{1}{7}+\frac{1}{8}}} = 0.1727$$

$$|t| = 0.1727$$

(viii) Conclusion

Here $|t| = 0.1727 < 2.16$, we reject the null hypothesis H_0 at 5% level of significance.

There is no significant difference between the two means.

CHI-SQUARE (χ^2) DISTRIBUTION

8. Using the data given in the following table to test at 1% level significance whether a person's ability in Mathematics is independent of his/her interest in Statistics.

			Ability in Mathematics	
		Low	**Average**	**High**
Interest in Statistics	**Low**	**63**	**42**	**15**
	Average	**58**	**61**	**31**
	High	**14**	**47**	**29**

Solution:

Table of expected frequencies.

	Low	Average	High	Total

Low	63	42	15	120
Average	58	61	31	150
High	14	47	29	90
Total	135	150	75	360

$\frac{135 \times 120}{360} = 45$	$\frac{150 \times 120}{360} = 50$	$\frac{75 \times 120}{360} = 25$	120
$\frac{135 \times 150}{360} = 56.25$	$\frac{150 \times 150}{360} = 62.5$	$\frac{75 \times 150}{360} = 31.25$	150
$\frac{135 \times 90}{360} = 33.75$	$\frac{150 \times 90}{360} = 37.5$	$\frac{75 \times 90}{360} = 18.75$	90
135	150	75	360

Calculated Ψ^2

Observed frequency(O)	**Expected frequency(E)**	**(O-E)**	$\frac{(O-E)^2}{E}$
63	45	324	7.5
42	50	64	1.28
15	25	100	4.00
58	56.25	3.0625	0.05
61	62.5	2.25	0.04

31	31.25	0.0625	0.002
14	33.75	390.0625	11.56
47	37.5	90.25	2.41
29	18.75	105.0625	5.60
			32.142

Now $\Psi^2 = \sum \frac{(O-E)^2}{E} = \mathbf{32.142}$

$\Psi^2 = 32.14$

Tab Ψ^2 fort d.f at 5% level is 9.488

Since Cal $\Psi^2 > Tab\ \Psi^2 \quad \therefore we\ reject\ H_0$.

Hence ability in Mathematics and interest in statistics are depended.

9. Fit a binomial distribution for the following data and also test the goodness of fit.

X:	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
F(X):	**5**	**18**	**28**	**12**	**7**	**6**	**4**	**80**

Solution:

Given n=2

(i) The parameter is χ^2

(ii) H_0:Binomial is good fit

(iii) H_1:Binomial is not a good fit

(iv) Level of significance $\propto = 0.0$

(v) The test statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$

(vi) Reject H_0 if $\chi^2 > {\chi^2}_{0.05}$(from χ^2table)

(vii) Computation :

$$p(x) = nC_X p^x q^{n-x}$$

$$E = \frac{80}{7} = 11$$

$$np = 2.4$$

$$6p = 2.4$$

$$p = 0.4, q = 0.6$$

Observed frequency(O)	Expected frequency(E)	$\frac{(O-E)^2}{E}$
5	11	3.27
18	11	4.46
28	11	26.27
12	11	0.99
7	11	1.46
6	11	2.27
4	11	4.46

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 6.17$$

$$n = 2$$

$$\chi^2_{0.05} = 5.98$$

if $\chi^2 > \chi^2_{0.05}$ Reject H_0

10. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory?

Solution:

Given A, B, C, D in the ration 9: 3: 3: 1

					Total
E_i:	900	300	300	100	1600
O_i:	882	313	287	118	1600

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{18^2}{900} + \frac{13^2}{300} + \frac{13^2}{300} + \frac{18^2}{100} = 4.73$$

$$\sum E_i = \sum O_i\ , \quad d.f = 4 - 1 = 3$$

H_0: The experiment supports the theory

$$Cal \quad \chi^2 = 4.73$$

Table χ^2 3 d.f=7.82

$$Cal \quad \chi^2 < \text{Table } \chi^2$$

So we accept H_0

F-TEST

11. A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight (gms)

Diet A: 5 6 8 1 12 4 3 9 6 10

Diet B: 2 3 6 8 10 1 2 8

Does it show superiority of diet A over diet B?

Solution:

Given $n_1 = 10, n_2 = 8$

x_1	${x_1}^2$	x_2	${x_2}^2$
5	25	2	4
6	36	3	9
8	64	6	36
1	1	8	64
12	144	10	100
4	16	1	1
3	9	2	4
9	81	8	64
6	36		
10	100		
64	512	40	282

$$\bar{x}_1 = \frac{\sum x_1}{n} = \frac{64}{10} = 6.4 \quad \bar{x}_2 = \frac{\sum x_2}{n} = \frac{40}{8} = 5$$

$$S_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{512}{10} - (6.4)^2 = 10.24$$

$$S_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{282}{8} - 25 = 10.25$$

$$S_1^2 < S_2^2$$

(i) The parameter is $\sigma_1^2 \& \sigma_2^2$.

(ii) Null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ (there is no significant difference).

(iii) Alternative hypothesis $H_0: \sigma_1^2 \neq \sigma_2^2$

(iv) Level of significance $\propto = 0.05$. Degree of freedom $(v_1) = 9, d.f\ (v_2) = 7\ ie., F_{(9,7)} = 3.29$

(v) Accept H_0 if calculated F $< F_{(9,7)} = 3.29$ (from table 'F')

(vi) **Computation:** The test statistic is $F = \frac{S_2^2}{S_1^2} = \frac{10.25}{10.24} = 1.0009$

(vii) Conclusion

Here $|F| = 1.0009 < 3.29$, so we accept the null hypothesis H_0.

We conclude that the two samples have come from populations with equal variances.

12. Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables.

Sample I	**18**	**13**	**12**	**15**	**12**	**14**	**16**	**14**	**15**
Sample II	**16**	**19**	**13**	**16**	**18**	**13**	**15**		

Do the estimates of the population variance differ significantly at 5% level ?

Solution:

Given $n_1 = 9, n_2 = 7$

x_1	x_1^2	x_2	x_2^2
18	324	16	256
13	169	19	361
12	144	13	169
15	225	16	256
12	144	18	324
14	196	13	169
16	256	15	225
14	196		
15	225		
129	1879	110	1760

$$\bar{x}_1 = \frac{\sum x_1}{n} = \frac{129}{9} = 14.3333 \bar{x}_2 = \frac{\sum x_2}{n} = \frac{110}{7} = 15.7143$$

$$S_1{}^2 = \frac{\sum x_1{}^2}{n_1} - (\overline{x_1})^2 = \frac{1879}{9} - (14.3333)^2 = 3.3342$$

$$S_2{}^2 = \frac{\sum x_2{}^2}{n_2} - (\overline{x_2})^2 = \frac{1760}{7} - (15.7143)^2 = 4.4894$$

$$S_1{}^2 < S_2{}^2$$

(i) The parameter is $\sigma_1{}^2 \& \sigma_2{}^2$.

(ii) Null hypothesis $H_0: \sigma_1{}^2 = \sigma_2{}^2$ (there is no significant difference).

(iii) Alternative hypothesis $H_0: \sigma_1{}^2 \neq \sigma_2{}^2$

(iv) Level of significance $\propto = 0.05$. Degree of freedom $(v_1) = 8, d.f\ (v_2) = 7\ ie., F_{(6,8)} = 3.58$

(v) Accept H_0 if calculated F $< F_{(6,8)} = 3.58$ (from table 'F')

(vi) **Computation:** The test statistic is $F = \frac{S_2{}^2}{S_1{}^2} = \frac{4.4894}{3.3342} = 1.3464$

(vii) Conclusion

Here $|F| = 1.3464 < 3.58$, so we accept the null hypothesis H_0.

We conclude that the difference is not significant.

13. Two random samples gave the following results:

Samples	Size	Sample mean	Sum of squares of deviation from the mean
1	10	15	90
2	12	14	108

Examine whether the samples come from the same normal population.

Solution:

A normal population has two parameters namely the mean μ and the variance σ^2. If we want to test samples from the same normal population, we have to test

(i) the equality of population variances (using F-test)

(ii) the equality of population means (using t-test)

Since t-test assumes $\sigma_1{}^2 = \sigma_2{}^2$ we shall first apply F-test and then t-test.

(i) <u>F-test</u>

Given $n_1 = 10,\ n_2 = 12, \overline{x_1} = 15, \overline{x_2} = 14$

$$S_1^{\ 2} = \sum \frac{(x-\bar{x})^2}{n_1 - 1} = \frac{90}{9} = 10$$

$$S_2^{\ 2} = \sum \frac{(x-\bar{x})^2}{n_2 - 1} = \frac{108}{11} = 9.8181$$

$$S_1^{\ 2} > S_2^{\ 2}$$

(i) The parameter of interest is $\sigma_1^{\ 2}$ and $\sigma_2^{\ 2}$

(ii) $H_0 : \sigma_1^{\ 2} = \sigma_2^{\ 2}$

(iii) $H_1 : \sigma_1^{\ 2} \neq \sigma_2^{\ 2}$

(iv) $\alpha = 0.05$, $d.f\ (v_1) = n_1 - 1 = 9$ $d.f\ (v_1) = n_2 - 1 = 11$

(v) The test statistic is $F = \frac{S_1^{\ 2}}{S_2^{\ 2}}$

(vi) Reject H_0 if F > 2.90 (from table 'F')

(vii) **Computation:** $F = \frac{10}{9.8182} = 1.019$

(viii) **Conclusion:** Here F=1.019< 2.90, so we accept H_0 at 5% level of significance.
[Note: If F-test failed, then t-test should not be used]

(ii) <u>t-test</u>

Given $n_1 = 10,\ n_2 = 12, S_1^{\ 2} = 10, S_2^{\ 2} = 9.8181$

$$S^2 = \frac{n_1 s_1^{\ 2} + n_2 s_2^{\ 2}}{n_1 + n_2 - 2} = \frac{100 + 117.82}{10 + 12 - 2} = 10.9$$

(i) The parameter is $\mu_1 \& \mu_2$.

(ii) Null hypothesis $H_0 : \mu_1 = \mu_2$ (there is no significant difference).

(iii) Alternative hypothesis $H_1 : \mu_1 \neq \mu_2$

(iv) Level of significance $\propto = 0.05$. Degree of freedom=$n_1 + n_2 - 2 = 20$

(v) The test statistic is $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

(vi) Reject if $|t| > 2.086$[from table 't' we get t=2.086] at 5% level of significance.

(vii) Computation:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{1}{1.3472}$$

$$= 0.707$$

(viii) Conclusion

Here $|t| = 0.707 < 2.086$, so we accept the null hypothesis H_0 at 5% level of significance.

Hence the difference is not significant.

Final conclusion: From the above two test, we can conclude that the two sample drawn from the same normal population.

UNIT-II

DESIGN OF EXPERIMENTS

PART – A

1. **Define Analysis of variance (ANOVA)**

   Solution:

   Analysis of variance (ANOVA) is a technique that will enable us to test for the significance of the difference among more than two sample means.

2. **State the basic principles of design of experiments.**

   Solution:

   There are three basic principles of experimental design. They are (i) Randomization, (ii) Replication, (iii) Local control (error control)

3. **What is the aim of the design of experiments?**

   Solution:

The main aim of the design of experiments is to control the extraneous variables and hence to minimize the experimental error so that the result of the experiments could be attributed only to the experimental variable.

4. **State the assumptions involved in ANOVA.**

Solution:

For the validity of the F-test in ANOVA, the following assumptions are made:

(i) The observations are independent.

(ii) Parent population from which observations are taken in normal and

(iii) Various treatment and environmental effects are additive in nature.

5. **What are the uses of ANOVA?**

Solution :

Analysis of variance is useful , for example , for determining

(I)Which of various training methods produces the fastest learning record.

(ii) Whether the effects of some fertiliserson the yields are significantly different,

(iii) Whether the mean qualities of outputs of various machines differ significantly etc. In fact this technique finds application in nearly every type of experimental design in natural sciences as well as in social sciences.

COMPLETELY RANDOMIZED DESIGN

6. **Compare one-way classification model with two-way classification model.**

Solution:

One way	Two way
1. We cannot test two sets of hypothesis	1. Two sets of hypothesis can be tested.
2. Data are classified according to one factor	2. Data are classified according to the different factor.

7. **What is a completely randomized design.**

Solution:

In a completely randomized design the treatments are given to the experimental units by a procedure of random allocation. It is used when the units are homogeneous.

8. **State any two advantages of a Completely Randomized Experimental Design.**

Solution:

(i) It is easy to lay out the design.

(ii) It allows for complete flexibility. Any number of factor classes and replications may be used.

(iii) The statistical analysis is relatively simple, even if we do not have the same number of replicates for each factor class or if the experimental errors are not the same from class to class of this factor.

(iv) The method of analysis remains when data are missing or rejected and the loss of information due to missing data is smaller than any other design.

9. **Define 2^2 factorial design .**

Solution:

A two-factor factorial design is an experimental design in which data is collected for all possible combinations of the levels of the two factors of interest.

10. **Why 2×2 Latin square is not possible? Explain.**

Solution:

In Latin square, the formula for degrees of freedom for residual (SSE) is d.f=$(n-1)(n-2)$

Substituting $n = 2,\ d.f = 0$

$$MSE = \infty$$

$\therefore$ 2×2 Latin square is not possible

PART – B

COMPLETELY RANDOMIZED DESIGN (C.R.D) (OR) [ONE WAY CLASSIFICATION]

1. The following table shows the lives in hours of four brands of electric lamps

Brand A	**1610**	**1610**	**1650**	**1680**	**1700**	**1720**	**1800**	
B	**1580**	**1640**	**1640**	**1700**	**1750**			
C	**1460**	**1550**	**1600**	**1620**	**1640**	**1660**	**1740**	**1820**
D	**1510**	**1520**	**1530**	**1570**	**1600**	**1680**		

Perform an analysis of variance test the homogeneity of the mean lives of the four brands of lamps.

Solution:

H_0: There is no significant difference between the four brands

H_1: There is significant difference between the four brands

Subtract 1600 and then divided by 10 we get

X_1 A	X_2 B	X_3 C	X_4 D	**Total**	X_1^2	X_2^2	X_3^2	X_4^2

1	-2	-14	-9	**-24**	1	4	196	81
1	4	-5	-8	**-8**	1	16	25	64
5	4	0	-7	**2**	25	16	0	49
8	10	2	-3	**17**	64	100	4	9
10	15	4	0	**29**	100	225	16	0
12	-	6	8	**26**	144	-	36	64
20	-	14	-	**34**	400	-	196	-
-	-	22	-	**22**	-	-	484	-
57	**31**	**29**	**-19**	**98**	**735**	**361**	**957**	**267**

Step 1: $N = 26$

Step 2: T=98

$\frac{T^2}{N} = \frac{9604}{26} = 369.39$

TSS=$\sum {X_1}^2 + \sum {X_2}^2 + \sum {X_3}^2 + \sum {X_4}^2 - \frac{T^2}{N}$

$= 753 + 361 + 957 + 267 - 369.39$

$= 1950.61$

SSC=$\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$ N_1 →Number of elements in each column

$$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{6} + \frac{(-19)^2}{6} - 369.39$$

$= 452.25$

$SSE = TSS - SSC$

$= 1950.61 - 452.25 = 1498.36$

Step 6: ANOVA

Source of variaions	**Sum of squares**	**Degree of freedom**	**Mean squares**	**Variance-ratio**	**Table value at 5% level**
Between Columns	SSC=452.25	C-1=4-1=3	MSC=$\frac{SSC}{C-1}$ = $\frac{452.25}{3}$ = 150.75	$F_c = \frac{MSC}{MSE}$ $= \frac{150.75}{68.11}$ = 2.21 Since $\frac{MSE}{MSC} <$ 1	$F_C(3{,}22)$ = 3.05
Error	SSE=1498.36	N-C=26-4=22	MSE= $\frac{SSE}{N-C}$ = $\frac{1498.36}{22}$ = 68.11		

Step 7: Conclusion: Cal $F_c <$ Tab F_C

$\therefore$ So we accept H_0

RANDOMIZED BLOCK DESIGN (R.B.D) (or) [TWO WAY CLASSIFICATION]

2. **A set of data involving four "four tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respect expect the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data.**

Weight gain of baby chicks fed on different feeding materials composed of tropical feed stuffs.

						Total T_i
A	**55**	**49**	**42**	**21**	**52**	**219**
B	**61**	**112**	**30**	**89**	**63**	**355**
C	**42**	**97**	**81**	**95**	**92**	**407**

D	169	137	169	85	154	714
Grand Total						G=1695

Solution :

Null hypothesisH_0:

(i) H_0: There is no significant difference between treatments(columns)

(ii) H_1: There is no significant difference between stuffs.(rows)

Code the data by subtracting 50 from each value.

Stuffs	Treatments									
	X_1	X_2	X_3	X_4	X_5	**Total**	${X_1}^2$	${X_2}^2$	${X_3}^2$	${X_4}^2$	${X_5}^2$
(Y_1) =**A**	5	-1	-8	-29	2	-31	25	1	64	841	4
(Y_2) =**B**	11	62	-20	39	13	105	121	3844	400	1521	169
(Y_3) =**C**	-8	47	31	45	42	157	64	2209	961	2025	1764
(Y_4) =**D**	119	87	119	35	104	464	14161	7569	14161	1225	10816
Total	127	195	122	90	161	695(T)	14371	13623	15586	5612	12753

Step: 1

N=20.

Step: 2

T =695

Step: 3

$$\frac{T^2}{N} = \frac{(695)^2}{20} = 24151.25$$

Step: 4

$$\text{TSS}=\sum {X_1}^2 + \sum {X_2}^2 + \sum {X_3}^2 + \sum {X_4}^2 + \sum {X_5}^2 - \frac{T^2}{N}$$

$$= 14371 + 13623 + 15586 + 5612 + 12753 - 24151.25$$

$$= 37793.75$$

Step:5

SSC=$\frac{(\sum X_1)^2}{N_1}+\frac{(\sum X_2)^2}{N_1}+\frac{(\sum X_3)^2}{N_1}+\frac{(\sum X_4)^2}{N_1}+\frac{(\sum X_5)^2}{N_1}-\frac{T^2}{N}$; N_1 →Number of elements in each column

$$=\frac{(127)^2}{4}+\frac{(195)^2}{4}+\frac{(122)^2}{4}+\frac{(90)^2}{4}+\frac{(161)^2}{4}-24151.25$$

$$=4032.25+9506.25+3721+2025+6480.25-24151.25$$

$$=1613.50$$

Step: 6

SSR= $\frac{(\sum Y_1)^2}{N_2}+\frac{(\sum Y)^2}{N_2}+\frac{(\sum Y_3)^2}{N_2}+\frac{(\sum Y_4)^2}{N_2}-\frac{T^2}{N}$ N_2 →Number of elements in each row

$$=\frac{(-31)^2}{5}+\frac{(105)^2}{5}+\frac{(157)^2}{5}+\frac{(464)^2}{5}-24151.25$$

$$=192.2+2205+4929.8+43059.2-24151.25$$

$$=26234.95$$

SSE= $\boldsymbol{TSS-SSC-SSR}$

$$=37793.75-1613.50-26234.95$$

$$=9945.3$$

Step: 7 ANOVA TABLE

Source of variaions	Sum of squares	Degree of freedom	Mean squares	F-ratio	Table value at 5% level
Between Block(Columns)	SSC=1613.50	c-1=5-1=4	MSC=$\frac{SSC}{d.f}=$ 403.375	$F_c=\frac{MSE}{MSC}$ $=2.055$	$F_c(12,4)=5.91$
Between Varieties(Rows)	SSR=26234.95	r-1=4-1=3	MSR= $\frac{SSR}{d.f}=$ $\frac{26234.95}{3}$	$F_R=\frac{MSR}{MSE}$ $=10.55$	$F_R(3,12)=3.49$
Residual	SSE=9945.3	(N-c-r+1)=12	MSE=$\frac{SSE}{12}=\frac{9945.3}{12}=$ 828.775	-	

Step: 8 Conclusion:

Cal $F_c<Table\ F_c$ so we accept H_0

Cal $F_R > Table\ F_R$ so we reject H_0

3. Carry out ANOVA (Analysis of variance) for the following

		A	B	C	D
Workers	**1**	**44**	**38**	**47**	**36**
	2	**46**	**40**	**52**	**43**
	3	**34**	**36**	**44**	**32**
	4	**43**	**38**	**46**	**33**
	5	**38**	**42**	**49**	**39**

Solution:

Null hypothesis H_0:

(i) The mean productivity is the same for four different machines

(ii) The 5 men do not differ with respect to mean productivity.

Code the data by subtracting 40 from each value.

The coded data is

Workers	**Machine Type**				**Total**				
	X_1	X_2	X_3	X_4		${X_1}^2$	${X_2}^2$	${X_3}^2$	${X_4}^2$
Y_1	4	-2	7	-4	**5**	16	4	49	16
Y_2	6	0	12	3	**21**	36	0	144	9
Y_3	-6	-4	4	-8	**-14**	36	16	16	64
Y_4	3	-2	6	-7	**0**	9	4	36	49
Y_5	-2	2	9	-1	**8**	4	4	81	1
Total	**5**	**-6**	**38**	**-17**	**T=20**	**101**	**28**	**326**	**139**

Step: 1

N =20

Step: 2

T=20

Step: 3

Correction factor (C.F)= $\frac{T^2}{N} = \frac{400}{20} = 20$

Step: 4

TSS=$\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N}$

$= 101 + 28 + 326 + 139 - 20$

$= 574$

SSC=$\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} + \frac{(\sum X_5)^2}{N_1} - \frac{T^2}{N}$ N_1 →Number of elements in each column

$$= \frac{(5)^2}{4} + \frac{(-6)^2}{4} + \frac{(38)^2}{4} + \frac{(-17)^2}{4} - C.F$$

$= 5 + 7.2 + 288.8 + 57.8 - 20$

$= 338.8$

SSR= $\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$ N_2 →Number of elements in each row

$$= \frac{(5)^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - C.F$$

$= 6.25 + 110.25 + 49 + 16 - 20$

$= 161.5$

$\boldsymbol{SSE = TSS - SSC - SSR}$

$= 574 - 338.8 - 161.5$

$= 73.7$

ANOVA TABLE

Source of variations	Sum of squares	Degree of freedom(D.f)	Mean squares	F-ratio	Table value at 5% level
Between Block(Columns)	SSC=338.8	$C - 1$=4-1=3	MSC=$\frac{SSC}{D.f}$ =	$F_c = \frac{MSC}{MSE}$ = 18.38	$F_c(3,12)$ = 3.49

			$\frac{338.8}{3} = 112.933$		
Between Varieties(Rows)	SSR=161.5	$R-1$=5-1=4	MSR= $\frac{SSR}{D.f}$ = $\frac{161.5}{4} = 40.375$	$F_R = \frac{MSR}{MSE}$ $= 6.574$	$F_R(4{,}12)$ $= 3.26$
Residual	SSE=73.7	$(C-1)(R-1)$=12	MSE=$\frac{SSE}{D.f}$ = $\frac{73.7}{12} = 6.142$	-	

Step: 8 Conclusion

(i) Table $F_c(3{,}12)$ *at* 5% *level* $= 3.49$

Calculated value $F_C = 18.38 > F_T = 3.49$ Reject H_0

∴ Mean productivity is not the same for the four different types of machines.

(ii) Table $F_R(4{,}12)$ *at* 5% *level* $= 3.26$

Calculated value $F_C = 6.58$ >$F_T = 3.26$ Reject H_0

∴ The workers differ with respect to mean productivity.

4. A company appoints 4 salesmen A, B, C and D and observes their sales in 3 seasons, summer, winter and monsoon. The figures are given in the following table:

season	**Salesmen**			
	A	**B**	**C**	**D**
Summer	**45**	**40**	**28**	**37**
Winter	**43**	**41**	**45**	**38**
Monsoon	**39**	**39**	**43**	**41**

Carry out an analysis of variances.

Solution:

Null hypothesis H_0: (i)The salesman do not differ significantly in their performance.

(ii)There is no significant difference between the seasons.

Code the data by subtraction 43 from each value to simplify calculations.

Seasons	Salesmen				Total	$X^2{}_1$	$X^2{}_2$	$X^2{}_3$	$X^2{}_4$
	X_1	X_2	X_3	X_4					
Summer	-2	-3	15	6	**16**	4	9	225	36
Winter	0	2	-2	5	**5**	0	4	4	25
Monsoon	4	4	0	2	**10**	16	16	0	4
Total	**2**	**3**	**13**	**13**	**31**	**20**	**29**	**229**	**65**

Step 1: $N = 12$

Step 2: T=31

Step 3: $\frac{T^2}{N} = \frac{(31)^2}{12} = 80$

Step 4:

TSS=$\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N}$

$= 20 + 29 + 229 + 65 - 80$

$= 263$

Step 5:

SSC=$\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$ N_1 →Number of elements in each column

$$= \frac{(2)^2}{3} + \frac{(3)^2}{3} + \frac{(13)^2}{3} + \frac{(13)^2}{3} - 80$$

$= 37.6$

Step 6:

SSR=$\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} - \frac{T^2}{N}$ N_2 →Number of elements in each row

$$= \frac{(16)^2}{4} + \frac{(5)^2}{4} + \frac{(10)^2}{4} - 80$$

$= 15.25$

$SSE = TSS - SSC - SSR$

$= 263 - 37.6 - 15.25 = 210.15$

Step 7: ANOVA TABLE

Source of variations	Sum of squares	Degree of freedom	Mean squares	Variance-ratio	Table value at 5% level
Between Block(Columns)	SSC=37.67	c-1=4-1=3	MSC=$\frac{SSC}{c-1}$ $=\frac{37.67}{3}=12.56$	$F_c=\frac{MSE}{MSC}$ $=2.79$	$F_C(6,3)=8.94$
Between Varieties(Rows)	SSR=15.25	r-1=3-1=2	MSR= $\frac{SSR}{r-1}$ $=\frac{15.25}{2}=7.63$	$F_R=\frac{MSE}{MSR}$ $=4.587$	$F_R(6,2)=19.33$
Residual	SSE=210.15	$(C-1)(R-1)$=6	MSE=$\frac{SSE}{N-C-r+1}$ $=\frac{210.15}{6}=35$	-	

Step 8: Conclusion:

Cal $F_c <$ Tab F_C

$\therefore$ So we accept H_0

5. **Four varieties A, B, C, D of a fertilizer are tested in a randomized block design with 4 replication. The plot yields in pounds are as follows:**

Column Row	1	2	3	4
1	A(12)	D(20)	C(16)	B(10)
2	D(18)	A(14)	B(11)	C(14)
3	B(12)	C(15)	D(19)	A(13)
4	C(16)	B(11)	A(15)	D(20)

Analyze the experimental yield.

Solution:

Null hypothesis H_0: Four varieties are similar

Alternative hypothesis H_1: Four varieties are not similar.

Calculation of correction factor

Variety	Block				Total of varieties				
	1 (X_1)	2 (X_2)	3 (X_3)	4 (X_4)		${X^2}_1$	${X^2}_2$	${X^2}_3$	${X^2}_4$
A	12	14	15	13	**54**	144	196	225	169
B	12	11	11	10	**44**	144	121	121	100
C	16	15	16	14	**61**	256	225	256	196
D	18	20	19	20	**77**	324	400	361	400
Total	**58**	**60**	**61**	**57**	**236**	**868**	**942**	**963**	**865**

Step 1: $N = 16$

Step 2: T=236

Step 3: $\frac{T^2}{N} = \frac{(236)^2}{16} = 3481$

Step 4:

TSS=$\sum {X_1}^2 + \sum {X_2}^2 + \sum {X_3}^2 + \sum {X_4}^2 - \frac{T^2}{N}$

$= 868 + 942 + 963 + 865 - 3481$

$= 157$

Step 5:

SSC=$\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$ N_1 →Number of elements in each column

$$= \frac{(58)^2}{4} + \frac{(60)^2}{4} + \frac{(61)^2}{4} + \frac{(57)^2}{4} - 3481$$

$$= 841 + 900 + 930 + 812 - 3481 = 2$$

Step 6:

SSR=$\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$ N_2 →Number of elements in each row

$$= \frac{(54)^2}{4} + \frac{(44)^2}{4} + \frac{(61)^2}{4} + \frac{(77)^2}{4} - 3481$$

$$= 729 + 484 + 930.25 + 1482.25 - 3481$$

$$= 144.5$$

$$\boldsymbol{SSE = TSS - SSC - SSR}$$

$$= 157 - 2 - 144.5 = 10.5$$

Step 7: ANOVA TABLE

Source of variations	Sum of squares	Degree of freedom	Mean squares	Variance-ratio	Table value at 5% level
Between Varieties	$SSC = 144.5$	$C - 1 = 3$	$MSC = \frac{SSC}{C-1} = \frac{144.5}{3}$ $= 48.17$	$F_C = \frac{MSC}{MSE}$ $= \frac{48.17}{1.22} = 39.48$	$F_C(3,9)$ $= 3.86$
Between Blocks	$SSR = 2$	$R - 1 = 3$	$MSR = \frac{SSR}{r-1} = \frac{2}{3}$ $= 0.67$	$F_R = \frac{MSE}{MSR} = \frac{1.22}{0.67}$ $= 1.82$	$F_R(9,3)$ $= 8.81$
Residual	$SSE = 11$	$(C-1)(R-1)$ $= 9$	MSE $= \frac{SSE}{(C-1)(R-1)}$ $= \frac{11}{9} = 1.22$	-	

Step 8: Conclusion:

Cal $F_c >$ Tab F_C ; Cal $F_R <$ Tab F_R

$\therefore$ So we reject H_0

Hence four varieties are not similar. But the varieties are similar along block wise.

LATIN SQUARE DESIGNS

6. Analyze the variance in the Latin square of yields (in kgs) of paddy where P,Q,R,S denote the different methods of cultivation:

S122 P121 R123 Q122

Q124 R123 P122 S125

P120 Q119 S120 R121

R122 S123 Q121 P122

Examine whether different method of cultivation have significantly different yields.

Solution:

Null hypothesis H_0: There is no significant difference between rows, between columns and treatments.

Let us take 120 as origin for simplifying the calculations.

Table I

	(X_1)	(X_2)	(X_3)	(X_4)	**Total**	${X^2}_1$	${X^2}_2$	${X^2}_3$	${X^2}_4$
Y_1	2	1	3	2	**8**	4	1	9	4
Y_2	4	3	2	5	**14**	16	9	4	25
Y_3	0	-1	0	1	**0**	0	1	0	1
Y_4	2	3	1	2	**8**	4	9	1	4
Total	**8**	**6**	**6**	**10**	**30**	**24**	**20**	**14**	**34**

Step 1: $N = 16$

Step 2: T=30

Step 3: $\frac{T^2}{N} = \frac{(30)^2}{16} = 56.25$

Step 4:

TSS=$\sum {X_1}^2 + \sum {X_2}^2 + \sum {X_3}^2 + \sum {X_4}^2 - \frac{T^2}{N}$

$= 24 + 20 + 14 + 34 - 56.25$

$= 35.75$

Step 5:

SSC=$\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$ N_1 →Number of elements in each column

$$= \frac{(8)^2}{4} + \frac{(6)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 56.25$$

$$= 2.75$$

Step 6:

SSR$= \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$ N_2 →Number of elements in each row

$$= \frac{(8)^2}{4} + \frac{(14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - 56.25$$

$$= 24.75$$

Table II: To find SST:

					T
P	0	1	2	2	5
Q	4	-1	1	2	6
R	2	3	3	1	9
S	2	3	0	5	10

$$SST = \frac{(5)^2}{4} + \frac{(6)^2}{4} + \frac{(9)^2}{4} + \frac{(10)^2}{4} - 56.25$$

$$= 4.25$$

$$\boldsymbol{SSE = TSS - SSC - SSR - SST}$$

$$= 35.75 - 2.75 - 24.75 - 4.25 = 4$$

Step 7: ANOVA

Source of variaions	Sum of squares	Degree of freedom	Mean squares	Variance-ratio	Table value at 5% level
Between Column	SSC $= 2.75$	$n - 1 = 3$	$MSC = \frac{SSC}{n-1}$ $= 0.917$	$F_c = \frac{MSC}{MSE} = \frac{0.917}{0.667} = 1.375$	$F_C(3,6) = 4.76$

Between Row	$SSR = 24.75$	$n-1=3$	$MSR = \frac{SSR}{n-1} = 8.25$	$F_R = \frac{MSR}{MSE} = \frac{8.25}{0.667} = 12.369$	$F_R(3,6) = 4.76$
Between Treatments	$SST = 4.25$	$n-1=3$	$MST = \frac{SST}{n-1} = 1.417$	$F_T = \frac{MST}{MSE} = \frac{1.417}{0.667} = 2.124$	$F_T(3,6) = 4.76$
Error	$SSE = 4$	$(n-1)(n-2) = 6$	$MSE = \frac{SSE}{6} = 0.667$	$-$	
Total	$TSS = 35.75$				

Step 8: Conclusion:

$$\text{Cal } F_c < \text{Tab } F_C$$

$$\text{Cal } F_R > \text{Tab } F_R$$

$$\text{Cal } F_T < \text{Tab } F_T$$

There is a significant difference between rows.

But there is no significant difference between columns and treatments.

7. **The following is a Latin square design when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of obtain and scale.**

A	**105**	**B**	**95**	**C**	**125**	**D**	**115**
C	**115**	**D**	**125**	**A**	**105**	**B**	**105**
D	**115**	**C**	**95**	**B**	**105**	**A**	**115**
B	**95**	**A**	**135**	**D**	**95**	**C**	**115**

Solution:

Subtract 100 and then divided by 5 we get

A	1	B	-1	C	5	D	3
C	3	D	5	A	1	B	1
D	3	C	-1	B	1	A	3
B	-1	A	7	D	-1	C	3

Table I

$\boldsymbol{Y_n}$	(X_1)	(X_2)	(X_3)	(X_4)	**Total**	$X^2{}_1$	$X^2{}_2$	$X^2{}_3$	$X^2{}_4$
Y_1	1	-1	5	3	**8**	1	1	25	9
Y_2	3	5	1	1	**10**	9	25	1	1
Y_3	3	-1	1	3	**6**	9	1	1	9
Y_4	-1	7	-1	3	**8**	1	49	1	9
Total	**6**	**10**	**6**	**10**	**32**	**20**	**76**	**28**	**28**

Step 1: $N = 16$

Step 2: T=32

Step 3: $\frac{T^2}{N} = \frac{(32)^2}{16} = 64$

Step 4:

TSS=$\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N}$

$= 20 + 76 + 28 + 28 - 64$

$= 88$

Step 5:

SSC=$\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$ N_1 →Number of elements in each column

$$= \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(10)^2}{4} - 64$$

$$= 9 + 25 + 9 + 25 - 64 = 4$$

Step 6:

SSR$=\frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$ N_2 →Number of elements in each row

$$= \frac{(8)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(8)^2}{4} - 64$$

$$= 16 + 25 + 9 + 16 - 64$$

$$= 2$$

Table II: To find SST:

					Total
A	1	1	3	7	12
B	-1	1	1	-1	0
C	5	3	-1	3	10
D	3	5	3	-1	10

$$SST = \frac{(12)^2}{4} + \frac{(0)^2}{4} + \frac{(10)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N}$$

$$= 36 + 0 + 25 + 25 - 64 = 22$$

$$\boldsymbol{SSE = TSS - SSC - SSR - SST}$$

$$= 88 - 4 - 2 - 22 = 60$$

Step 7: ANOVA

Source of variaions	Sum of squares	Degree of freedom	Mean squares	Variance-ratio	Table value at 5% level
Between Column	$SSC = 4$	$n - 1 = 3$	$MSC = \frac{SSC}{n-1} = 1.33$	$F_c = \frac{MSC}{MSE}$ $= \frac{10}{1.33} = 7.52$	$F_C(6,3) = 8.94$

Between Row	$SSR = 2$	$n-1=3$	$MSR = \frac{SSR}{n-1} = 0.67$	$F_R = \frac{MSR}{MSE}$ $= \frac{10}{0.67} = 14.9$	$F_R(6,3) = 8.94$
Between Treatments	$SST = 22$	$n-1=3$	$MST = \frac{SST}{n-1} = 7.33$	$F_T = \frac{MST}{MSE}$ $= \frac{10}{7.33} = 1.36$	$F_T(6,3) = 8.94$
Error	$SSE = 60$	$(n-1)(n-2)$ $= 6$	$MSE = \frac{SSE}{(n-1)(n-2)}$ $= 10$	$-$	
Total	$TSS = 88$	15			

Step 8: Conclusion: Cal $F_R >$ Tab F_R

Cal $F_c <$ Tab F_C

Cal $F_T <$ Tab F_T

There is a significant difference between rows as well as columns.

But there is no significant difference between treatments.

UNIT-III

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART – A

FIXED POINT ITERATION METHOD

1. **What is the order of convergence and the condition for convergence of fixed point iteration method?**

Sol:

Order of convergence: 1

Condition for convergence: $|\phi'(x)| < 1$

NEWTON'S METHOD (OR) NEWTON RAPHSON METHOD

2. **State the order of convergence and condition for convergence of Newton-Raphson method. (OR)**

Write the convergence condition and order of convergence for Newton-Raphson method.

Solution: Order of convergence is two.

Condition for convergence is $|f(x).f''(x)| < |f'(x)|^2$

3. **Find the smallest positive roots of the equation $x^3 - 2x + 0.5 = 0$**

Solution:

f(x) = $x^3 - 2x + 0.5$

$f'(x) = \mathbf{3x^2 - 2}$

$f(0) = 0.5(+ve)$

$f(1) = -0.5\ (-ve)$

Hence the roots lies between 0 and 1. Since the value of f(x) at x=0 is very close to zero than the value of f(x) at x=1, we can say that the root is very close to 0. Therefore we can assume that $x_0 = 0$ is the initial approximation to the root.

Newton's formula is

$$\boldsymbol{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Putting n=0 in (1), we get the first approximation x_1 to the root, given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{0.5}{-2}$$

$$x_1 = 0.25$$

Putting n=1 in (1), we get the second approximation x_2 to the root, given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.25 - \frac{(0.25)^3 - 2(0.25) + 0.5}{3(0.25)^2 - 2}$$

$$= 0.25 - \frac{0.0156}{-1.8125} = 0.2586$$

$$\boldsymbol{x_2 = 0.2586}$$

Putting n=2 in (1), we get the third approximation x_3 to the root, given by

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.2586 - \frac{(0.2586)^3 - 2(0.2586) + 0.5}{3(0.2586)^2 - 2}$$

$$\boldsymbol{x_3 = 0.2586}$$

Hence the smallest positive root is $\mathbf{0.2586.}$

4. Derive the formula to find the value of $1/N$ where $N \neq 0$, using Newton Raphson method.

Solution:

Let $x = \frac{1}{N}$

$$N = \frac{1}{x}$$

$$\frac{1}{x} - N = 0$$

$f(x) = \frac{1}{x} - N$; $f'(x) = -\frac{1}{x^2}$

The Newton's formula is $\boldsymbol{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$

$$= x_n - \frac{\left(\frac{1}{x_n} - N\right)}{\left(-\frac{1}{x_n^{\,2}}\right)}$$

$$= x_n + \left(\frac{1}{x_n} - N\right) X \;\; x_n^{\,2}$$

$$= x_n + x_n - x_n^{\,2}N$$

$$\boldsymbol{x_{n+1}} = x_n(2 - Nx_n)$$

5. Arrive a formula to find the value of $\sqrt[3]{N}$ where $N \neq 0$, using Newton-Raphson method.

Solution:

Let $x = \sqrt[3]{N}$

$$x^3 = N$$

$$x^3 - N = 0$$

$f(x) = x^3 - N$; $f'(x) = 3x^2$

By Newton-Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{{x_n}^3 - N}{3{x_n}^2} = \frac{3{x_n}^3 - {x_n}^3 + N}{3{x_n}^2}$$

$$= \frac{1}{3}\left[\frac{2{x_n}^3 + N}{{x_n}^2}\right]$$

$$= \frac{1}{3}\left[2x_n + \frac{N}{{x_n}^2}\right], n = 0,1,2, \dots \dots$$

GAUSSIAN ELIMINATION AND GAUSS – JORDON METHODS

6. Give two direct methods to solve a system of linear equation.

Solution:

* Gauss Elimination Method

* Gauss Jordon Method.

7. Compare Gauss – Jacobi and Gauss – Sedial method.

Solution:

S.No	Gauss – Jacobi method	Gauss – Sedial method
1.	Convergence rate is slow	The rate of convergence of Gauss – Seidal method is fast, roughly twice that of Gauss – Jacobi
2.	Indirect method	Indirect method
3.	condition for convergence is the coefficient matrix is diagonally dominant	Condition for convergence is the co-efficient matrix is diagonally dominant.

8. Solve 3x + 2y = 4, 2x – 3y = 7 by Gauss elimination method.

Solution:

Given 3x + 2y = 4

2x – 3y = 7

The given system is equivalent to

$$\begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Here $[A, B] = \left(\begin{matrix} 3 & 2 \\ 2 & -3 \end{matrix}\middle|\begin{matrix} 4 \\ 7 \end{matrix}\right)$

$$= \left(\begin{matrix} 3 & 2 \\ 0 & -13 \end{matrix}\middle|\begin{matrix} 4 \\ 13 \end{matrix}\right) R_2 \leftrightarrow 3R_2 - 2R_1$$

This is an upper triangular matrix

Using backward substitution method

$-13y = 13$

$y = -1$

$3x + 2y = 4$

$3x - 2 = 4$

$3x = 6$

$x = 2$

Hence the solution is x = 2 and y = -1

9. Which iterative method converges faster for solving linear system of equations? Why?

Sol:

Gauss Seidal method is solving for linear system of equations converge faster. In this method the rate of convergence is roughly twice as fast as that of Gauss- Jacobi’s method.

10.Write the uses of power method?

Sol:

To find the numerically largest eigen value of a given matrix.

11. Find the dominant eigen value and eigenvector of the matrix $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}$ by power method.

Solution: $Let\ X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$AX_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.43 \\ 1 \end{pmatrix} = 7X_2$$

$$AX_2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0.43 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.43 \\ 5.29 \end{pmatrix} = 5.29\begin{pmatrix} 0.46 \\ 1 \end{pmatrix} = 5.29X_3$$

$$AX_3 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0.46 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.46 \\ 5.38 \end{pmatrix} = 5.38\begin{pmatrix} 0.46 \\ 1 \end{pmatrix} = 5.38X_4$$

Hence the dominant eigen value=5.38

The corresponding eigen vector= $\begin{pmatrix} 0.46 \\ 1 \end{pmatrix}$.

PART – B

FIXED POINT ITERATION METHOD

1. Using fixed point iteration method to find the positive root of the equation $\cos x - 3x + 1 = 0$.

Sol:

Given that $\cos x = 3x - 1$

Let $f(x) = cosx - 3x + 1 = 0$

$f(0) = 1 - 0 + 1 = 2 = +ve$

$f(1) = cos1 - 3 + 1 = -1.4597 = -ve$

So, a root lies between 0 and 1

The given equation may be written as

$$x = \frac{1}{3}(1 + cosx) = g(x)$$

$$g(x) = \frac{1}{3}(1 + cosx)$$

$$g'(x) = -\frac{1}{3}sinx$$

$$|g'(x)| = \frac{1}{3}sinx$$

$$|g'(0)| = 0 < 1$$

$$|g'(1)| = \frac{1}{3}\sin 1 = 0.2804 < 1$$

So, the method can be applied.

Let $x_0 = 0.6$

$$x_1 = \frac{1}{3}[1 + cosx_0] = \frac{1}{3}[1 + \cos(0.6)] = 0.60845$$

$$x_2 = \frac{1}{3}[1 + cosx_1] = 0.60684$$

$$x_3 = \frac{1}{3}[1 + cosx_2] = 0.60715$$

$$x_4 = \frac{1}{3}[1 + cosx_5] = 0.60709$$

$$x_5 = \frac{1}{3}[1 + cosx_4] = 0.60710$$

$$x_6 = \frac{1}{3}[1 + cosx_5] = 0.60710$$

Here $x_5 = x_6 = 0.60710$

Hence, the better approximate root is 0.60710

2. Solve $e^x - 3x = 0$ by method of fixed point iteration.

Sol:

G.T: $e^x - 3x = 0$

Let $f(x) = e^x - 3x$

$f(0) = e^0 - 3(0) = 1(+ve)$

$f(1) = e^1 - 3(1) = e - 3(-ve)$

Therefore the root lies between 0 &1.

The given equation is of the form, $x = \frac{e^x}{3} = g(x)$

$$g'(x) = \frac{e^x}{3}$$

$$|g'(x)| = \frac{e^x}{3}$$

$$|g'(0)| = \frac{1}{3} < 1$$

$$|g'(1)| = \frac{e}{3} < 1$$

Hence the condition is satisfied

Let us assume $x_0 = 0.6$

$$x_1 = \frac{e^{x_0}}{3} = \frac{1}{3}e^{0.6} = 0.6074$$

$$x_2 = \frac{e^{x_1}}{3} = \frac{1}{3}e^{0.6074} = 0.6119$$

$$x_3 = \frac{e^{x_2}}{3} = 0.6146$$

`

$$x_4 = \frac{e^{x_3}}{3} = 0.6163$$

$$x_5 = \frac{e^{x_4}}{3} = 0.6174$$

$$x_6 = \frac{e^{x_5}}{3} = 0.6180$$

$$x_7 = \frac{e^{x_6}}{3} = 0.6184$$

$$x_8 = \frac{e^{x_7}}{3} = 0.6187$$

$$x_9 = \frac{e^{x_8}}{3} = 0.6188$$

$$x_{10} = \frac{e^{x_9}}{3} = 0.6189$$

$$x_{11} = \frac{e^{x_{10}}}{3} = 0.6190$$

$$x_{12} = \frac{e^{x_{11}}}{3} = 0.6190$$

$\therefore\ x_{11} = x_{12} = 0.6190$ correct to 4 decimal places.

Hence, the better approximate root is 0.6190.

NEWTON'S METHOD (OR) NEWTON RAPHSON METHOD

3. Solve the equation $x\log_{10} x = 1.2$ using Newton-Raphson method.

Solution:

Let $f(x) = x\log_{10}x - 1.2 \Rightarrow f'(x) = x \times \frac{1}{x}\log_{10}e + \log_{10}x$

$$f'(x) = \log_{10}e + \log_{10}x$$

$f(0) = 0\ \log_{10}(0) - 1.2 = -1.2 = -ve$

$f(1) = 1\ \log_{10}(1) - 1.2 = -1.2 = -ve$

$f(2) = 2\ \log_{10}(2) - 1.2 = -0.598 = -ve$

$f(3) = 3\ \log_{10}(3) - 1.2 = 0.231 = +ve$

Therefore the root lies between 2 & 3

$|f(2)| > |f(3)|$

Hence the root is nearer to 3choose $x_0 = 2.7$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.7 - \frac{f(2.7)}{f'(2.7)} = 1 - \left[\frac{2.7\, log_{10}(2.7) - 1.2}{log_{10}e + log_{10}2.7}\right] = 2.7 - \left[\frac{-0.035}{0.867}\right]$$

$x_1 = 2.740$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.740 - \frac{f(2.740)}{f'(2.740)} = 2.740 - \left[\frac{-0.006}{0.872}\right]$$

$x_2 = 2.741$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.741 - \frac{f(2.741)}{f'(2.741)} = 2.741 - \left[\frac{-.003}{0872}\right]$$

$x_3 = 2.741$

We observe that the root $x_2 = x_3 = 2.741$Correct to 3 decimal places. Hence the required root correct to three decimal places is 2.741

4. Find the real positive root of $3x - cos\, x - 1 = 0$ by Newton's method correct to 5 decimal places.

Solution :

Let $f(x) = 3x - cos\, x - 1$

$$f'(x) = 3 + sin\, x$$

$$f(0) = 0 - 1 - 1 = -2 = -ve$$

$$f(1) = 3 - cos\, 1 - 1 = 2 - cos\, 1 = 1.459698 = +ve$$

Therefore a root lies between 0 and 1.

$$|f(0)| > |f(1)|$$

Hence the root lies between 0 and 1.

$$Let\ \ x_0 = 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f^{'}(x_n)} \ldots \ldots \ldots (1)$$

Let n=0 in equation (1)

$$x_1 = x_0 - \frac{f(x_0)}{f^{'}(x_0)}$$

$$= 1 - \frac{f(1)}{f^{'}(1)}$$

$$= 1 - \left[\frac{3(1) - \cos(1) - 1}{3 + \sin(1)}\right]$$

$$= 1 - \left[\frac{1.45970}{3.84147}\right]$$

$$= 1 - 0.37998$$

$$x_1 = 0.62002$$

Let n=1 in equation (1)

$$x_2 = x_1 - \frac{f(x_1)}{f^{'}(x_1)}$$

$$= 0.62002 - \frac{f(0.62002)}{f^{'}(0.62002)}$$

$$= 0.62002 - \left[\frac{3(0.62002) - \cos(0.62002) - 1}{3 + \sin(0.62002)}\right]$$

$$x_2 = 0.60712$$

Let n=2 in equation (1)

$$x_3 = x_2 - \frac{f(x_2)}{f^{'}(x_2)}$$

$$= 0.60712 - \frac{f(0.60712)}{f^{'}(0.607102)}$$

$$= 0.60712 - \left[\frac{3(0.60712) - \cos(0.60712) - 1}{3 + \sin(0.60712)}\right]$$

$$x_3 = 0.60710$$

Let n=3 in equation (1)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.60710 - \frac{f(0.60710)}{f'(0.60710)}$$

$$= 0.60710 - \left[\frac{3(0.60710) - \cos(0.60710) - 1}{3 + \sin(0.60710)}\right]$$

$$x_3 = 0.60710$$

From x_2 and x_3 we findout the root is 0.60710 correct to five decimal places.

5. Interpret the Newton's iterative formula to calculate the reciprocal of N and hence find the value of 1/26.

Sol:

Let $x = \frac{1}{N}$

$$N = \frac{1}{x}$$

$$\frac{1}{x} - N = 0$$

$f(x) = \frac{1}{x} - N$; $f'(x) = -\frac{1}{x^2}$

The Newton's formula is $\boldsymbol{x_{n+1}} = \boldsymbol{x_n} - \frac{\boldsymbol{f(x_n)}}{\boldsymbol{f'(x_n)}}$

$$= x_n - \frac{\left(\frac{1}{x_n} - N\right)}{\left(-\frac{1}{x_n^{\,2}}\right)}$$

$$= x_n + \left(\frac{1}{x_n} - N\right) X \ \ x_n^{\,2}$$

$= x_n + x_n - x_n{}^2 N$

$= x_n(2 - N x_n)$

To find 1/26, take N=26

Let $x_0 = 0.04$

W.K.T $\boldsymbol{x_{n+1}} = x_n(2 - N x_n)$

$\boldsymbol{x_1} = x_0(2 - 26x_0)$

$= 0.04(2 - 26(0.04))$

$= 0.0384$

$\boldsymbol{x_2} = x_1(2 - 26x_1)$

$= 0.0384(2 - 26(0.0384))$

$= 0.0385$

$\boldsymbol{x_3} = x_2(2 - 26x_2)$

$= 0.0385(2 - 26(0.0385))$

$= 0.0385$

Here $\boldsymbol{x_2} = x_3 = 0.0385$

Hence the value of 1/26=0.0385

SOLUTION OF LINEAR SYSTEM BY GAUSSIAN ELIMINATION METHOD

6. Solve the system of equations using Gauss elimination method

$\boldsymbol{5x - 2y + z = 4;\ \ 7x + y - 5z = 8; 3x + 7y + 4z = 10}$ **.**

Solution:

The given system is equivalent to

$$\begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

$$[A, B] = \left(\begin{array}{ccc|c} 5 & -2 & 1 & 10 \\ 7 & 1 & -5 & 18 \\ 3 & 7 & 4 & 16 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 5 & -2 & 1 & 4 \\ 0 & -19 & 27 & -12 \\ 0 & -41 & -17 & -38 \end{array}\right) \quad \begin{matrix} R_2 \leftrightarrow 7R_1 - 5R_2 \\ R_3 \leftrightarrow 3R_1 - 5R_3 \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 5 & -2 & 1 & 4 \\ 0 & -19 & 27 & -12 \\ 0 & 0 & 1430 & 230 \end{array}\right) \quad R_3 \leftrightarrow 41R_2 - 19R_3$$

Use back substitution to find the solution to the system.

$$1430z = 230$$

$$z = 230/1430$$

$$z = 0.161$$

$$-19y + 27z = -12$$

$$-19y = -12 - 4.343$$

$$Y = 0.860$$

$$5x - 2y + z = 4$$

$$5x - 1.72 + 1.161 = 4$$

$$x = 1.112$$

Hence $\boldsymbol{x = 1.112}$, $\boldsymbol{y = 0.860}$, $\boldsymbol{z = 0.161}$.

7. Solve the following equations by Gauss elimination method:

$$\boldsymbol{2x + y + 4z = 12;\ \ 8x - 3y + 2z = 20; 4x + 11y - z = 33}$$

Solution:

The given system is equivalent to

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$[A,B]=\begin{pmatrix}2 & 1 & 4 & 12\\ 8 & -3 & 2 & 20\\ 1 & 4 & 9 & 33\end{pmatrix}$$

$$\sim\left(\begin{array}{ccc|c}2 & 1 & 4 & 12\\ 0 & 7 & 14 & 18\\ 0 & -9 & 9 & -9\end{array}\right) R_2\to 4R_1-R_2, R_3\to 2R_1-R_3$$

$$\sim\left(\begin{array}{ccc|c}2 & 1 & 4 & 12\\ 0 & 7 & 14 & 18\\ 0 & 0 & 189 & 189\end{array}\right) R_3\to 7R_3+9R_2$$

Use back substitution to find the solution to the system.

$$189z = 189$$

$$\boldsymbol{z = 1}$$

$$7y + 14z = 28 \;\Rightarrow 7y + 14 = 28 \Rightarrow 7\,y = 14$$

$$\boldsymbol{y = 2}$$

$$2x + y + 4z = 12 \Rightarrow 2x + 2 + 4 = 12$$

$$2x = 6$$

$$\boldsymbol{x = 3}$$

Hence $\boldsymbol{x = 3}$, $\boldsymbol{y = 2}$, $\boldsymbol{z = 1}$.

SOLUTION OF LINEAR SYSTEM BY GAUSS – JORDAN METHODS

8. Using the Gauss – Jordan method solve the following equations 10x + y + z = 12, 2x + 10y + z = 13, x + y +5z =7

Solution:

Given $10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 5z = 7$

Interchanging the first and the last equation then

$$x + y + 5z = 7$$

$$2x + 10y + z = 13$$

$$10x + y + z = 12$$

The given system is equivalent to

$$\begin{bmatrix} 1 & 1 & 5 \\ 2 & 10 & 1 \\ 10 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \\ 12 \end{bmatrix}$$

AX = B

Here $[A,\ B] = \left(\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{array}\right)$

Fix the pivot element row and make the other elements zero in the pivot element column.

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{array}\right) R_2 \leftrightarrow R_2 - 2R_1 \ \& \ R_3 \leftrightarrow R_3 - 10R_1$$

$$\sim \left(\begin{array}{ccc|c} 8 & 0 & 49 & 57 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 473 & 473 \end{array}\right) R_1 \leftrightarrow 8R_1 - R_2 \ \& \ R_3 \leftrightarrow 8R_3 + 9R_2$$

$$\sim \left(\begin{array}{ccc|c} 8 & 0 & 49 & 57 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) R_3 \leftrightarrow \frac{R_3}{473}$$

$$\sim \left(\begin{array}{ccc|c} 8 & 0 & 0 & 8 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array}\right) R_1 \leftrightarrow R_1 - 49R_3 \ \& \ R_2 \leftrightarrow R_2 + 9R_3$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) R_1 \leftrightarrow \frac{R_1}{8} \ \& \ R_2 \leftrightarrow \frac{R_2}{8}$$

Therefore the solution is $\boldsymbol{x = 1, y = 1, z = 1}$

9. Using the Gauss – Jordan method solve the following equations $\boldsymbol{2x - y + 3z = 8}$, $\boldsymbol{-x + 2y + z = 4, 3x + y - 4z = 0}$

Solution:

Given $2x - y + 3z = 8$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$

The given system is equivalent to

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$AX = B$$

Here $[A,\ B] = \left(\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right)$

Fix the pivot element row and make the other elements zero in the pivot element column.

$$\sim\left(\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 5 & -17 & -24 \end{array}\right) R_2 \leftrightarrow 2R_2 + R_1 \ \ \& \ R_3 \leftrightarrow 2R_3 - 3R_1$$

$$\sim\left(\begin{array}{ccc|c} 6 & 0 & 14 & 40 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -76 & -152 \end{array}\right) \begin{array}{l} R_1 \leftrightarrow 3R_1 + R_2 \\ R_3 \leftrightarrow 3R_3 - 5R_2 \end{array}$$

$$\sim\left(\begin{array}{ccc|c} 6 & 0 & 14 & 40 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & 1 & 2 \end{array}\right) R_3 \leftrightarrow \frac{R_3}{-76}$$

$$\sim\left(\begin{array}{ccc|c} 6 & 0 & 0 & 12 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 1 & 2 \end{array}\right) R_1 \leftrightarrow R_1 - 14R_3 \ \ \& \ R_2 \leftrightarrow R_2 - 5R_3$$

$$\sim\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right) \quad R_1 \leftrightarrow \frac{R_1}{6} \ \& \ R_2 \leftrightarrow \frac{R_2}{3}$$

Therefore the solution is $\boldsymbol{x = 2, y = 2, z = 2}$

<u>GAUSS – JACOBI METHOD AND GAUSS – SEDIAL METHOD</u>

10. Solve the system of equation by Gauss – Sedial method correct to 4 decimal places

20x + y –2 z = 17, 3x + 20y -z = -18, 2x -3y +20z = 25

Solution: .

Given 20x + y –2 z = 17

3x + 20y -z = -18

2x -3y +20z = 25

As the coefficient matrix is diagonally dominant solving for x, y, z we get

$$x = \frac{1}{20}[17 - y + 2z], y = \frac{1}{20}[-18 - 3x + z], z = \frac{1}{20}[25 - 2x + 3y]$$

Let the initial value be y=0, z=0

Iteration	$x = \left[\frac{17 - y + 2z}{20}\right]$	$y = \left[\frac{-18 - 3x + z}{20}\right]$	$z = \left[\frac{25 - 2x + 3y}{20}\right]$
1	0.85	-1.0275	1.0109
2	1.0025	-0.9998	0.9998
3	1.0000	-1.0000	1.0000
4	1	-1	1

Hence x = 1, y = -1, z = 1.

11. Solve the system of equation by Gauss – Seidel method 28x +4 y – z = 32, x + 3y +10z = 24, 2x +17y +4z = 35.

Solution: .

Given $28x + 4y - z = 32$

$x + 3y + 10z = 24$

$2x + 17y + 4z = 35$

As the coefficient matrix is diagonally dominant solving for x, y, z we get

$$x = \frac{1}{28}[32 - 4y + z]$$

$$y = \frac{1}{17}[35 - 2x - 4z]$$

$$z = \frac{1}{20}[24 - x - 3y]$$

Let the initial value be y=0, z=0

Iteration	$x = \left[\frac{32 - 4y + z}{28}\right]$	$y = \left[\frac{35 - 2x - 4z}{17}\right]$	$z = \left[\frac{24 - x - 3y}{20}\right]$

Let the initial value be y=0, z=0

1	1.1429	1.9244	1.8084
2	0.9325	1.5236	1.8497
3	0.9913	1.5070	1.8488
4	0.9936	1.5069	1.8486
5	0.9936	1.5069	1.8486

Hence x =0.9936 , y = 1.5069, z = 1.8486

EIGEN VALUE OF A MATRIX BY POWER METHOD

12. Find the largest Eigen value and the corresponding Eigen vector of $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{6} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{3} \end{bmatrix}$ using power method. Using $\boldsymbol{x_1} = (\mathbf{1} \quad \mathbf{0} \quad \mathbf{0})^{\boldsymbol{T}}$ as initial vector.

Solution:

Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an approximate eigen value.

$$AX_1 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1\, X_2$$

$$AX_2 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = 7\, X_3$$

$$AX_3 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.5714 \\ 1.8572 \\ 0 \end{bmatrix} = 3.5714 \begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = 3.5714\, X_4$$

$$AX_4 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.12 \\ 2.04 \\ 0 \end{bmatrix} = 4.12 \begin{bmatrix} 1 \\ 0.4951 \\ 0 \end{bmatrix} = 4.12\, X_5$$

$$AX_5 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4951 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.9706 \\ 1.9902 \\ 0 \end{bmatrix} = 3.9706 \begin{bmatrix} 1 \\ 0.5012 \\ 0 \end{bmatrix} = 3.9706\, X_6$$

$$AX_6 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5012 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.0072 \\ 2.0024 \\ 0 \end{bmatrix} = 4.0072 \begin{bmatrix} 1 \\ 0.4997 \\ 0 \end{bmatrix} = 4.0072\ X_6$$

$$AX_7 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4997 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.9982 \\ 1.9994 \\ 0 \end{bmatrix} = 3.9982 \begin{bmatrix} 1 \\ 0.5000 \\ 0 \end{bmatrix} = 3.9982\ X_8$$

$$AX_8 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = 4\ X_9$$

$$AX_9 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}$$

Therefore Dominant eigen value =4; corresponding eigen vector is (1, 0.5, 0)

13. Find , by power method, the largest Eigen value and the corresponding Eigen vector of a matrix $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ with initial vector $(1\ 1\ 1)^T$.

Solution:

Let $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ be an arbitrary initial eigen vector.

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 13 \end{bmatrix} = 13 \begin{bmatrix} 0.231 \\ 0.692 \\ 1 \end{bmatrix} = 13X_2$$

$$AX_2 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.231 \\ 0.692 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.307 \\ 6.077 \\ 12.537 \end{bmatrix} = 12.537 \begin{bmatrix} 0.104 \\ 0.485 \\ 1 \end{bmatrix} = 12.537X_3$$

$$AX_3 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.104 \\ 0.485 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.559 \\ 5.282 \\ 11.836 \end{bmatrix} = 11.836 \begin{bmatrix} 0.047 \\ 0.485 \\ 1 \end{bmatrix} = 11.836X_4$$

$$AX_4 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.047 \\ 0.485 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.385 \\ 5.033 \\ 11.737 \end{bmatrix} = 11.737 \begin{bmatrix} 0.033 \\ 0.429 \\ 1 \end{bmatrix} = 11.737X_5$$

$$AX_5 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.033 \\ 0.429 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.32 \\ 4.957 \\ 1 \end{bmatrix} = 11.683 \begin{bmatrix} 0.027 \\ 0.424 \\ 1 \end{bmatrix} = 11.683X_6$$

$$AX_6 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.027 \\ 0.424 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.299 \\ 4.929 \\ 11.669 \end{bmatrix} = 11.669 \begin{bmatrix} 0.026 \\ 0.422 \\ 1 \end{bmatrix} = 11.669X_7$$

$$AX_7 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.026 \\ 0.422 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.299 \\ 4.922 \\ 11.662 \end{bmatrix} = 11.662\begin{bmatrix} 0.025 \\ 0.422 \\ 1 \end{bmatrix} = 11.662X_8$$

$$AX_8 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.025 \\ 0.422 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.291 \\ 4.919 \\ 11.663 \end{bmatrix} = 11.663\begin{bmatrix} 0.025 \\ 0.422 \\ 1 \end{bmatrix}$$

Therefore, the dominant eigenvector is $\begin{bmatrix} 0.025 \\ 0.422 \\ 1 \end{bmatrix}$, eigenvalue is 11.663.

UNIT-IV

INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

PART-A

LAGRANGE'S INTERPOLATION

1. **Write down the Lagrange's Interpolation formula.**

   Solution:

   Let y = f(x) be a function which takes the values $y_0, y_1, y_2, \ldots\ldots y_n$ corresponding to $x_0, x_1, x_2, \ldots. x_n$

   Then Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1$$

$$+ \ldots\ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$

2. **Find the second degree polynomial through the points (0,2),(2,1),(1,0) using Lagrange's formula.**

   Solution:

   We use Lagrange's interpolation formula

$$y = f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

$$= \frac{(x-2)(x-1)}{(0-2)(0-1)}.2 + \frac{(x-0)(x-1)}{(2-0)(2-1)}.1 + \frac{(x-0)(x-2)}{(1-0)(1-2)}.0$$

$$= x^2 - 3x + 2 + \frac{1}{2}(x^2 - x) = \frac{1}{2}(2x^2 - 6x + 4 + x^2 - x)$$

$$y = \frac{1}{2}(3x^2 - 7x + 4)$$

DIVIDED DIFFERENCES

3. Distinguish between interpolation and extrapolation.

Solution:

Interpolation	Extrapolation
To find the values of a function inside a given range is interpolation.	To find the values of a function outside a given range is extrapolation.

4. Find the divided difference of f(x) which takes the values 1, 4, 40, 85 with arguments 0, 1, 3, 4

Solution:

The divided difference table is as follows

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$

0	1			
		$\frac{4-1}{1-0} = 3$		
			$\frac{18-3}{3-0} = 5$	
1	4			
		$\frac{40-4}{3-1} = 18$		$\frac{6.75-5}{4-0} = 0.44$
			$\frac{45-18}{4-0} = 6.75$	
3	40			
		$\frac{85-40}{4-3} = 45$		
4	85			

5. **Find the divided differences of $f(x) = x^3 + x + 2$ for the arguments 1, 3 , 6, 11.**

Solution:

$$f(1) = 1^3 + 1 + 2 = 4$$

$$f(3) = 3^3 + 3 + 2 = 32$$

$$f(6) = 6^3 + 6 + 2 = 224$$

$$f(11) = 11^3 + 11 + 2 = 1344$$

The divided difference table is as follows

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$

1	4			
		$\frac{32-4}{3-1}=14$		
			$\frac{64-14}{6-1}=10$	
3	32			
		$\frac{224-32}{6-3}=64$		$\frac{20-10}{11-1}=1$
			$\frac{224-64}{11-3}=20$	
6	224			
		$\frac{1344-224}{11-6}=224$		
11	1344			

<u>NEWTON'S FORWARD AND BACKWARDINTERPOLATION</u>

6. Derive Newton's backward interpolation formula using operator method.

(OR) State Newton's backward formula for interpolation.

State Newton's backward difference formula.

Solution:

$$y(x) = y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \cdots$$

Where $v = \frac{x-x_n}{h}$

7. Derive Newton's forward interpolation formula using equal intervals .

Solution:

$$y_n = f(x_0 + n\text{h}) = y_0 + n\Delta y_n + \frac{n(n-1)}{2!}\Delta^2 y_n + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_n + + \cdots$$

8. Find the first and second divided difference with arguments $\boldsymbol{a, b, c}$ of the function $\boldsymbol{f(x) = \frac{1}{x}}$

Solution:

If $f(x) = \frac{1}{x} \;\Rightarrow f(a) = \frac{1}{a}$

$$f(a,b) = \Delta\left[\frac{1}{a}\right] = \frac{\frac{1}{b}-\frac{1}{a}}{b-a} = -\frac{1}{ab} \qquad \left[\because f(x_0,x_1) = \frac{f(x_1)-f(x_0)}{x_0-x_1}\right]$$

$$f(a,b,c) = \frac{f(b,c)-f(a,b)}{c-a} = \frac{-\frac{1}{bc}-\frac{1}{ab}}{c-a} = \frac{1}{abc}\left[\frac{c-a}{c-a}\right] = \frac{1}{abc}$$

$\therefore \qquad \Delta^2\left[\frac{1}{a}\right] = \frac{1}{abc}$

DIFFERENTION USING INTERPOLATION FORMULA

9. Write down the expression for $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ *at* $\boldsymbol{x = x_n}$ by Newton's backward difference formula.

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n - \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

NUMERICAL INTEGRATION BY TRAPEZOIDAL METHOD

10. State Trapezoidal rule to evaluate $\int_a^b f(x)dx$.

Solution:

$$\int_a^b f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})]$$

11. Taking $h = 0.5$, evaluate $\int_1^2 \frac{dx}{1+x^2}$ using Trapezoidal rule.

Solution:

Here $y(x) = \frac{1}{1+x^2}$

Length of the interval = 1

x	:	1	1.5	2
$y = \frac{1}{1+x^2}$	:	0.5	0.3077	0.2

$h = 0.5$

By Trapezoidal rule

Trapezoidal rule

$$= \frac{h}{2}[sum\ of\ the\ first\ and\ last\ ordinates]$$

$$+ 2[sum\ of\ the\ remaining\ ordinates]$$

$$\int_1^2 \frac{dx}{1+x^2} = \frac{h}{2}[(0.5 + 0.2) + 2(0.3077)]$$

$$\int_1^2 \frac{dx}{1+x^2} = \frac{0.5}{2}[0.7 + 0.6154]$$

$$\int_1^2 \frac{dx}{1+x^2} = \frac{0.5}{2}[1.3154] = 0.3289$$

12. Using Trapezoidal rule, evaluate $\int_0^{\pi} \sin x\,dx$ by dividing the range into 6 equal parts.

Solution:

Given: $\int_0^{\pi} \sin x\,dx$

Range = b - a = $\pi - 0 = \pi$

Here $h = \frac{\pi}{6}$

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π
$y = \sin x$	**0.0**	**0.5**	**0.866**	**1**	**0.866**	**0.5**	**0**

(i) By Trapezoidal Rule:

$$\int_a^b f(x)dx = \frac{h}{2}(A + 2B)$$

$$= \frac{\pi}{12}[(0+0) + 2(0.5 + 0.866 + 1 + 0.8666 + 0.5)]$$

= 0.6220 π

13. Evaluate $\int_{\frac{1}{2}}^{1} \frac{1}{x}dx$ by Trapezoidal rule, dividing the range into 4equal parts.

Solution:

Here, $h = \frac{1-\frac{1}{2}}{4} = \frac{1}{8}$; $y = \frac{1}{x}$

x:	$1/2 = 4/8$	$5/8$	$6/8$	$7/8$	$8/8$
$f(x)$: $1/x$	$8/4$	$8/5$	$8/6$	$8/7$	$8/8$

A= Sum of the first and last ordinates $= \frac{8}{4} + \frac{8}{8} = 3$

B= Sum of the remaining ordinates =8/5+8/6+8/7=856/210

$$\therefore \int_{\frac{1}{2}}^{1} \frac{1}{x} dx = \frac{h}{2}[A + 2B] = \frac{1}{16}\left(3 + \frac{856 \times 2}{210}\right) = 0.6971$$

14. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule.

Solution:

Here $y(x) = \frac{1}{1+x^2}$

Length of the interval = 1

x	:	0	0.2	0.4	0.6	0.8	1
$y = \frac{1}{1+x^2}$	:	1	0.96154	0.86207	0.73529	0.60976	0.5

$h = 0.2$

By Trapezoidal rule

Trapezoidal rule

$$= \frac{h}{2}[sum\ of\ the\ first\ and\ last\ ordinates]$$

$$+ 2[sum\ of\ the\ remaining\ ordinates]$$

$$\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$\int_0^1 \frac{dx}{1+x^2} = \frac{0.2}{2}[(1 + 0.5) + 2(0.96154 + 0.86207 + 0.9412 + 0.73529 + 0.60976)]$$

$$\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{0.2}{2}[7.83732] = 0.783732 \quad \ldots\ldots..(1)$$

By actual integration,

$$\int_0^6 \frac{dx}{1+x^2} = (tan^{-1}x)_0^1 = tan^{-1}1 - tan^{-1}0 = \frac{\pi}{4} \quad \ldots\ldots(2)$$

$From\ (1)\&\ (2)$ $\qquad \frac{\pi}{4} = 0.783732$

$\pi = 3.13493(approximately$

NUMERICAL INTEGRATION BY SIMPSON'S 1/3 AND 3/8 RULES

15. State Simpson's one-third rule.

Solution:

Simpson's one third rule is

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) \quad + 2(y_2 + y_4 + \cdots + y_{n-2})]$$

16. State Trapezoidal rule for evaluating $\int_a^b \int_c^d f(x,y)\ dx\ dy$**.**

Solution:

$$I = \frac{hk}{4}[(Sum \text{ of values of f at Four corners}) + 2(\text{Sum of the values of f at remaining nodes on the boundary}) + 4\left(\text{sum of values of } f \text{ at interior nodes}\right)$$

PART-B

LAGRANGE'S INTERPOLATION

1. Find the interpolation polynomial f(x) by Lagrange's formula and hence find f(3) for (0,2),(1,3),(2,12)and (5,147). (OR)

Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x	0	1	2	5
$f(x)$	2	3	12	147

Solution:

By Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$y = f(x) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)$$

$$+\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$= \frac{(x-1)(x-2)(x-5)}{(-10)}(2) + \frac{x(x-2)(x-5)}{4}(3)$$

$$+\frac{(x-1)(x-5)}{-6}(12) + \frac{x(x-1)(x-2)}{60}(147)$$

Put x = 3 we get

$$y = f(3) = \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{3(3-2)(3-5)}{4}(3)$$

$$+\frac{3(3-1)(3-5)}{-6}(12) + \frac{3(3-1)(3-2)}{60}(147)$$

$$= \frac{2(-2)}{(-10)}(2) + \frac{3(-2)}{4}(3) + \frac{3(2)(-2)}{(-6)}(12) + \frac{3(2)}{60}(147)$$

$$y = f(3) = \frac{4}{10}(2) - \frac{6}{4}(3) + 2(12) + \frac{1}{10}(147) = \frac{8}{10} - \frac{18}{4} + 24 + \frac{147}{10}$$

$$f(3) = 35$$

2. **Use Lagrange's formula to construct a polynomial which takes the values** $f(0) = -12, f(1) = 0, f(3) = 6$ **and** $f(4) = 12$. **Hence find** $f(2)$.

Solution:

By Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y = f(x) = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + 0$$

$$+ \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}(6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}(12)$$

$$= \frac{(x-1)(x-3)(x-4)}{(-12)}(-12) + \frac{x(x-1)(x-4)}{(-6)}(6)$$

$$+ \frac{x(x-1)(x-3)}{12}(12)$$

$$= (x-1)(x-3)(x-4) - x(x-1)(x-4) + x(x-1)(x-3)$$

$$= (x-1)[x^2 - 3x - 4x + 12 - x^2 + 4x + x^2 - 3x]$$

$$= (x-1)(x^2 - 6x + 12)$$

$$= x^3 - 6x^2 + 12x - x^2 + 6x - 12$$

$$f(x) = x^3 - 7x^2 + 18x - 12$$

$$\therefore f(2) = 2^3 - 7(2)^2 + 18(2) - 12 = 4$$

$$\therefore f(2) = 4$$

DIVIDED DIFFERENCES

3. Determine $f(x)$ as a polynomial in x for the following data, using Newton's divided difference formula. Also find $f(3)$

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

Solution:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-124}{-1-(-4)}=-404$			
-1	33		$\frac{-28-(-404)}{0-(-4)}=94$		
		$\frac{5-33}{0-(-1)}=-28$			
				$\frac{10-94}{2-(-4)}=-14$	
			$\frac{2-(-28)}{2-(-1)}=10$		
0	5				
		$\frac{9-5}{2-0}=2$			$\frac{13+14}{5-(-4)}=3$
				$\frac{88-10}{5-(-1)}=13$	
			$\frac{442-2}{5-0}=88$		
2	9				
		$\frac{1335-9}{5-2}=442$			
5	1335				

By Newton's divided difference interpolation formula,

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \ (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3)$$

$$\Lsh \cdots \ldots \ldots \ldots .(1)$$

Here $f(x_0) = 1245, f(x_0, x_1) = -404, f(x_0, x_1, x_2) = 94, f(x_0, x_1, x_2, x_3) = -14$ & $f(x_0, x_1, x_2, x_3, x_4) = 3$,

Hence we using this formula in equation (1) we get

$$f(x) = 1245 + (x + 4)(-404) + (x + 4)(x + 1)(94) + \ (x + 4)(x + 1)(x - 0)(-14)$$

$$+ \ (x + 4)(x + 1)(x - 0)(x - 2)(3)$$

$$= 1245 - 404x - 1616 + 94x^2 + 470x + 376 - 14x^3 - 70x^2 - 56x$$

$$+ 3x[x^3 - 2x^2 + 5x^2 - 10x + 4x - 8\,]$$

$$= -14x^3 + 24x^2 + 10x + 5 + 3x[x^3 \mp 3x^2 - 6x - 8\,]$$

$$= -14x^3 + 24x^2 + 10x + 5 + 3x^4 + 9x^3 - 18x^2 - 24x$$

$$\therefore \quad f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

$$\Rightarrow \quad f(3) = 3 \times 3^4 - 5 \times 3^3 + 6 \times 3^2 - 14 \times 3 + 5 = 125$$

$$\therefore \qquad f(3) = 125$$

4. Use Newton's divided difference formula find f(9) given the values (5,150), (7,392), (13,2366) and (17,5202)

Solution:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$

5	150			
		$\frac{392-150}{7-5}=121$		
			$\frac{329-121}{13-5}=26$	
7	392			
		$\frac{2366-392}{13-7}=329$		
				$\frac{38-26}{17-5}=1$
			$\frac{709-329}{17-7}=38$	
13	2366			
		$\frac{5202-2366}{17-13}=709$		
17	5202			

By Newton's divided difference formula

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$$

$$+ (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3) \dots\dots (1)$$

$$= 150 + (x - 5)(121) + (x - 5)(x - 7)(26) + (x - 5)(x - 7)(x - 13)(1)$$

$$f(9) = 150 + (9 - 5)(121) + (9 - 5)(9 - 7)(26) + (9 - 5)(9 - 7)(9 - 13)(1)$$

$$f(9) = 150 + 484 + 192 - 32$$

$f(9) = 794$

NEWTON'S FORWARD AND BACKWARD INTERPOLATION

5. Find a polynomial of degree two for the data by Newton's forward difference formula.

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
y	**1**	**2**	**4**	**7**	**11**	**16**	**22**	**29**

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		1	
		2		0
2	4		1	
		3		0
3	7		1	
		4		0
4	11		1	
		5		0
5	16		1	
		6		0
6	22		1	
		7		
7	29			

Here $x_0 = 0,\ y_0 = 1,\ h = 1$

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \ldots\ldots$$

Where $u = \frac{x-x_0}{h} = \frac{x-0}{1} = x \Rightarrow u = x$

$$y(x) = 1 + x(1) + \frac{x(x-1)}{2!}(1)$$

$$= 1 + x + \frac{x^2-x}{2} = \frac{2+2x+x^2-x}{2}$$

$y(x) = \frac{1}{2}[x^2 + x + 2]$ is the required polynomial.

6. Using Newton's forward interpolation formula, find the cubic polynomial which takes the following values .

X	**0**	**1**	**2**	**3**
Y	**1**	**2**	**1**	**10**

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0 0	y_0 1			
		$2-1=1(\Delta y_0)$		
x_1 1	y_1 2		$-1-1$ $=-2$ $(\Delta^2 y_0)$	
		$1-2=-1(\Delta y_1)$		$10+2$ $=12$ $(\Delta^3 y_0)$
x_2 2	y_2 1			
			$9+1$ $=10$ $(\Delta^2 y_1)$	
		$10-1=9(\Delta y_2)$		
x_3 3	y_3 10			

We will use forward difference formula

$$y(x)=y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\cdots$$

Where $u=\frac{x-x_0}{h}=\frac{x-0}{1}=x \Rightarrow u=x$

$$\therefore \quad y(x)=1+x(1)+\frac{x(x-1)}{2!}(-2)+\frac{x(x-1)(x-2)}{3!}(12)$$

$$=1+x-\frac{x^2-x}{2}(2)+\frac{x(x-1)(x-2)}{6}(12)$$

$$= 1 + x - x^2 + x + 2x(x-1)(x-2)$$

$$= 1 + x - x^2 + x + 2x(x^2 - 3x + 2)$$

$$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$$

$$= 1 + 6x - 7x^2 + 2x^3$$

$$\therefore \quad y(x) = 2x^3 - 7x^2 + 6x + 1$$

$$y(4) = P_3(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$$

$$= 2(64) - 7(16) + 24 + 1$$

$$= 41$$

7. **From the given table compute the value of sin 38°**

x	**0**	**10**	**20**	**30**	**40**
sin x	**0**	**0.17365**	**0.34202**	**0.5**	**0.64279**

Solution:

We form the difference table:

x	$Y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	(y_n)				
0	0	(Δy_0)			
		0.17365	$(\Delta^2 y_0)$		
10	0.17365		-0.00528	$(\Delta^3 y_0)$	
		0.16837		-0.00511	$(\Delta^4 y_0)$
20	0.34202		-0.01039		0.00031
		0.15798		-0.00487	$(\nabla^4 y_n)$

30	0.5		-0.01519	$(\nabla^3 y_n)$	
		0.14279	$(\nabla^2 y_n)$		
40	0.64279				
(x_n)	(y_n)				

We will use backward difference formula

$$y(x) = y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \cdots$$

Where $v = \frac{x-x_n}{h} = \frac{40-0.064279}{10} = -0.2$

$y(38°) = 0.64279 \; - 0.028 - 0.0127 + 0.0290 \quad y(38°) = 0.64249$

$\sin 38° = 0.61568$

DIFFERENTION USING INTERPOLATION FORMULA

8. Construct $\frac{dx}{dy}$ *and* $\frac{d^2 y}{d^2 x}$ **at** $x = 51$, **from the following data:**

X:	50	60	70	80	90
Y:	19.96	36.65	58.81	77.21	94.61

Solution:

Given x= 51, x_0 = 50 h = 60 – 50 = 10

$$u = \frac{x-x_0}{h} = \frac{51-50}{10} = 0.1$$

At x = 51, u = 0.1

Difference table

x	y = f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$

50	19.96				
		16.69			
60	36.65		5.47		
		22.16		-9.23	
70	58.81		-3.76		11.99
		18.40		2.76	
80	77.21		-1.00		
		17.40			
90	94.61				

W.K.T the Newton's forward difference formula is

$$f'(x) = \left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0.1} = \frac{1}{h}[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0$$

$$+\frac{(4u^3-18u^2+22u-6)}{3!}\Delta^4 y_0 + ...]$$

$$f'(51) = \left(\frac{dy}{dx}\right)_{u=0.1} = \frac{1}{10}[16.69 + \frac{(0.2-1)}{2}(5.47) + \left(\frac{(3(0.1)^2)-6(0.1)+2}{3!}\right)(-9.23) +$$

$$\left(\frac{(4(0.1)^3-18(0.1)^2)+22(0.1)-6}{24}\right)(11.99) + ...]$$

$$= \frac{1}{10}\left[16.69 - 2.188 - 2.1998 - 1.9863\right]$$

$$f'(51) = 1.0316$$

$$f''(x) = \left(\frac{d^2y}{dx^2}\right)_{u=0.1} = \frac{1}{h^2}[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{(6u^2-18u+11)}{12}\Delta^4 y_0 + ...]$$

$$f''(51) = \frac{1}{100}\left[5.47 + (0.1-1)(-9.23) + \frac{(6(0.1)^2-18(0.1)+11)}{12}(11.99)\right]$$

$$=\frac{1}{100}\left[5.47+8.307+9.2523\right]$$

$$f''(51)=0.2303$$

9. For the given data, find the first two derivative at x=1.1

x	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**
y	**7.989**	**8.403**	**8.781**	**9.129**	**9.451**	**9.750**	**10.031**

Solution:

The difference table is as follows

X	y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	7.989						
		0.414					
1.1	8.403		-0.036				
		0.378		0.006			
1.2	8.781		-0.030		-0.002		
		0.348		0.004		0.001	
1.3	9.129		-0.026		-0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		-0.023		0.002		
		0.299		0.005			
1.5	9.750		-0.018		0		
		0.281					
1.6	10.031						

$$f'(x)=\frac{dy}{dx}=\frac{1}{h}\left[\Delta y_0+\frac{(2u-1)}{2!}\Delta^2 y_0+\frac{(3u^2-6u+2)}{3!}\Delta^3 y_0+\cdots\right]\quad Where\ u=\frac{x-x_0}{h}$$

$$u = \frac{x - x_0}{h} = \frac{1}{1} = 1$$

$$\left(\frac{dy}{dx}\right)_{x=1.1} = \frac{1}{1}\left[0.414 + \frac{(2(1) - 1)}{2!}(-0.036) + \frac{(3(1\,) - 6(1) + 2)}{3!}(0.006) + \cdots\right]$$

$$\left(\frac{dy}{dx}\right)_{x=1.1} = 0.3950$$

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NUMERICAL INTEGRATION BY TRAPEZOIDAL METHOD

10. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by i) Trapezoidal rule ii) Simpson's rule. And compare the result with its actual integration value.

Solution:

Here $y(x) = \frac{1}{1+x^2}$

Let $h = 1$

x :0	1	2	3	4	5	6
y: 1	0.5	0.2	0.1	0.058824	0.038462	0.27027

We know that for Trapezoidal rule

$$\int_0^6 \frac{dx}{1 + x^2} = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$\int_0^6 \frac{dx}{1 + x^2} = \frac{1}{2}[(1 + 0.27027) + 2(0.5 + 0.2 + 0.1 + 0.058824 + 0.038462)]$$

$$\int_0^6 \frac{dx}{1 + x^2} = 1.41079950$$

We know that Simpson's one third rule is

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}[(y_0 + y_5) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$\int_0^6 \frac{dx}{1+x^2} = \frac{1}{3}[(0.5 + 0.027027) + 4(0.5 + 0.1 + 0.038462) + 2(0.2 + 0.58824)]$$

$$\int_0^6 \frac{dx}{1+x^2} = 1.28241$$

We know that Simpson's three – eight rule is

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3}{8}[(1 + 0.027027) + 3(0.5 + 0.2 + 0.058824 + 0.038462) + 2(0.1)]$$

$$\int_0^6 \frac{dx}{1+x^2} = 1.35708188$$

By actual integration,

$$\int_0^6 \frac{dx}{1+x^2} = (tan^{-1}x)_0^6 = tan^{-1}6 = 1.40564764$$

Conclusion:

Here the value by trapezoidal rule is closer to the actual value than the value by Simpson's rule.

11. Take $h = 0.05$, evaluate $\int_1^{1.3} \sqrt{x}\,dx$ using Trapezoidal rule and Simpson's three-eighth rule. Solution:

x	1	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.118	1.1402

We know that for Trapezoidal rule

$$\int_{1}^{1.3} \sqrt{x}\, dx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$\int_{1}^{1.3} \sqrt{x}\, dx = \frac{0.05}{2}[(1 + 1.1402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.118)]$$

$$\int_{1}^{1.3} \sqrt{x}\, dx = 0.3215$$

We know that Simpson's three – eight rule is

$$\int_{x_0}^{x_0+nh} f(x)\, dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$\int_{1}^{1.3} \sqrt{x}\, dx = \frac{3(0.05)}{8}[(1 + 1.1402) + 3(1.0247 + 1.0488 + 1.0954 + 1.118) + 2(1.0724)]$$

$$\int_{1}^{1.3} \sqrt{x}\, dx = 0.3215$$

DOUBLE INTEGRALS USING TRAPEZOIDAL AND SIMPSON'S RULES

12. Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{1}{x+y}\, dx\, dy$ by using Trapezoidal rule taking h=0.1 and k=0.1

Solution:

y\x	1	1.1	1.2	1.3	1.4
1	0.5000	0.4762	0.4545	0.4348	0.4167
1.1	0.4762	0.4545	0.4348	0.4167	0.4000
1.2	0.4545	0.4348	0.4167	0.4000	0.3846

I = $\frac{hk}{4}$ [(sum of values of f at the four corners)

+ 2 (sum of values of f at the remaining nodes on the boundary)

+ 4(sum of the values of f at the interior nodes)]

$$I = \frac{(0.1)(0.1)}{4}[(0.5000 + 0.4167 + 0.3846 + 0.4545)$$

$$+ 2(0.4762 + 0.4545 + 0.4348 + 0.4000 + 0.4000 + 0.4167 + 0.4348 + 0.4762)$$

$$+ 4(0.4545 + 0.4348 + 0.4167)]$$

$I = 0.0349$

13. Evaluate $\int_0^1 \int_0^1 \frac{1}{x+y+1} dx\, dy$ by using Trapezoidal rule taking h=0.5 and k=0.25

Solution:

	0	0.5	1
0	1	0.6667	0.5
0.25	0.8	0.5714	0.4444
0.5	0.6667	0.5	0.40
0.75	0.5714	0.4444	0.3636
1	0.50	.40	0.3333

I = $\frac{hk}{4}$ [(sum of values of f at the four corners)

+ 2 (sum of values of f at the remaining nodes on the boundary)

+ 4(sum of the values of f at the interior nodes)]

$I = \frac{(0.5)(0.25)}{4}[(1 + 0.5 + 0.3333 + 0.5) + 2(0.667 + 0.4444 + 0.40 + 0.3636 + 0.40 + 0.5714 +$

$0.6667 + 0.8) + 4(0.5714 + 0.5 + 0.4444)]$

$= 0.5319$

14. Evaluate $\int_1^3 \int_1^2 \frac{1}{xy} dx\, dy$ by using Trapezoidal rule taking h=0.5 and k=0.5

Solution:

	1	1.5	2
1	1	0.667	0.5
1.5	0.667	0.4444	0.3333
2	0.5	0.3333	0.25
2.5	0.4000	0.2667	0.2000
3	0.3333	4.5000	0.1667

I = $\frac{hk}{4}$ [(sum of values of f at the four corners)

+ 2 (sum of values of f at the remaining nodes on the boundary)

+ 4(sum of the values of f at the interior nodes)]

$I = \frac{(0.5)(0.5)}{4}[(1 + 0.5 + 0.3333 + 0.1667) + 2(0.667 + 0.3333 + 0.25 + 0.2 + 4.5 + 0.4 +$

$0.5 + 0.667) + 4(0.4444 + 0.3333 + 0.2667)]$

$I = 1.3258$

15. Evaluate $\int_0^{1/2} \int_0^{1/2} \frac{sin(xy)}{1+xy} dx\, dy$ using Simpson's rule with h=1/4=k

Solution:

Let $f(x, y) = \frac{sin(xy)}{1+x}$

The values of $f(x, y)$ at the nodal points are given in the following table

	0	1/4	½
0	0	0	0
¼	0	0.0588	0.1108
1/2	0	0.1108	0.1979

By Simpson's rule, I = $\frac{hk}{9}$ [(sum of values of f at the four corners)

+ 2 (sum of the values of f at the odd position on the boundary except the corners)

+ 4 (sum of the values of f at the even position on the boundary)

+ {4 (sum of the values of f at odd positions) + 8 (sum of the values of

f at even positions) on the odd row of the matrix except boundary rows}

+ {8 (sum of the values of f at the odd positions)+16 (sum of the

Values of f at the even position) on the even rows of the matrix}]

$$I = \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)}{9}[(0+0+0.1979+0)+4(0+0+0.1108+0.1108)+16(0.0588)]$$

$I = 0.0141$

16. Evaluate $\int_1^{1.4}\int_2^{2.4}\frac{1}{xy}dx\,dy$ by using Trapezoidal rule taking h=0.1 and k=0.1 and verify with actual integration .

Solution:

y\x	1	1.1	1.2	1.3	1.4
1	0.5	0.4762	0.4545	0.4348	0.4167
1.1	0.4545	0.4329	0.4132	0.3953	0.3788
1.2	0.4167	0.3968	0.3788	0.3623	0.3472
1.3	0.3846	0.3663	0.3497	0.3344	0.3205

1.4	0.3571	0.3401	0.3247	0.3106	0.2976

I = $\frac{hk}{4}$ [(sum of values of f at the four corners)

+ 2 (sum of values of f at the remaining nodes on the boundary)

+ 4(sum of the values of f at the interior nodes)]

$$I = \frac{(0.1)(0.1)}{4}[(0.5000 + 0.4167 + 0.3571 + 0.2976)$$

$$+ 2(0.3846 + 0.4167 + 0.4545 + 0.4762 + 0.4545 + 0.4348 + 0.3788 + 0.3472$$

$$+ 0.3205 + 0.3106 + 0.3247 + 0.3401)$$

$$+ 4(0.4329 + 0.4132 + 0.3953 + 0.3968 + 0.3788 + 0.3623 + 0.3663 + 0.3497$$

$$+ 0.3344)]$$

$$I = 0.0614$$

By actual integration:

$$\int_1^{1.4}\int_2^{2.4}\frac{1}{xy}dx\,dy = \left(\int_1^{1.4}\frac{1}{y}dy)\,(\int_2^{2.4}\frac{1}{x}dx\right)$$

$$= (log\,y)_1^{1.4}(log\,y)_2^{2.4}$$

$$= (log\,1.4)[log2.4 - log2]$$

$$= log(1.4)log(1.2)$$

$$\int_1^{1.4}\int_2^{2.4}\frac{1}{xy}dx\,dy = 0.0613$$

UNIT-V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

PART-A

TAYLOR SERIES METHOD

1. **Using Taylor series method fine $y(1.1)$ given that $y' = x + y, y(1) = 0$**

   Solution:

Given $y' = x + y$ and $x_0 = 1, y_0 = 0$

We know that Taylor series formula is

$$y_1 = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots$$

$y' = x + y$ $\qquad$ $y_0' = 1 + 0 = 1$

$y'' = 1 + y'$ $\qquad$ $y_0'' = 1 + 1 = 2$

$y''' = y''$ $\qquad$ $y_0''' = 2$

$$y_1 = 0 + (x-1) + \frac{(x-1)^2}{2}(2) + \frac{(x-1)^3}{6}(2)$$

$$y(1.1) = 0 + (1.1-1) + \frac{(1.1-1)^2}{2}(2) + \frac{(1.1-1)^3}{6}(2)$$

$$y_1 = y(1.1) = 0.1103$$

2. **Find $y(0.1)$ if $\frac{dy}{dx} = 1 + y, y(0) = 1$ using Taylor series method.**

   Solution:

Given $y' = 1 + y$ and $x_0 = 0, y_0 = 1$

We know that Taylor series formula is

$$y_1 = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots$$

$y' = 1 + y$	$y_0' = 1 + 1 = 2$
$y'' = y'$	$y_0'' = 2$
$y''' = y''$	$y_0''' = 2$

$$y_1 = 1 + (x-0)2 + \frac{(x-0)^2}{2}(2) + \frac{(x-0)^3}{6}(2) + \frac{(x-0)^4}{24}(2)$$

$$= 1 + 2x + x^2 + \frac{x^3}{3} + \frac{x^4}{12}$$

$$y(0.1) = 1 + 2(0.1) + (0.1)^2 + \frac{(0.1)^3}{3} + \frac{(0.4)^4}{12}$$

$$y_1 = y(0.1) = 1.2103$$

3. **State the advantages and disadvantages of the Taylor's series method.**

Solution:

The method gives a straight forward adaptation of classic calculus to develop the solution as an infinite series. It is a powerful single step method if we are able to find the successive derivatives easily.

If f(x,y) involves some complicated algebraic structures then the calculation of higher derivatives becomes tedious and the method fails.

EULER AND MODIFIED EULER METHOD

4. **State Euler's method to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.**

Solution:

$y_1 = y_0 + hf(x_0, y_0)$ where $n = 0,1,2 \ldots$

5. **State Modified Euler's method to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.**

Solution:

$$y_1 = y_0 + hf\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)$$

6. **Find $y(0.1)$ by using Euler's method given that $\frac{dy}{dx} = x + y, y(0) = 1$.**

Solution:

Given $y' = x + y, x_0 = 0, y_0 = 1$

By Euler's method

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 1 + (0.1)(0 + 1) = 1 + 0.1 = 1.2$$

$$y_1 = y(0.1) = 1.2$$

7. **Find $y(0.2)$ for the equation $y' = y + e^x$, given that $y(0) = 0$ by using Euler's method.**

Solution:

Given $y' = y + e^x, x_0 = 0, y_0 = 0, h = 0.2$

By Euler algorithm, $y_1 = y_0 + hf(x_0, y_0)$

$$= 0 + 0.2f(0,0)$$

$$= 0.2[0 + e^0] = 0.2$$

$$y(0.2) = 0.2$$

RUNGE-KUTTA METHOD FOR SOLVING FIRST AND SECOND ORDER EQUATIONS

8. State the fourth order Runge-Kutta algorithm.

Solution:

Let h denote the interval between equidistant values of x. if the initial values are (x_0, y_0), the first increment in y is computed from the formulas.

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$x_1 = x_0 + h, y_1 = y_0 + \Delta y$$

The increment in y in the second interval is computed in a similar manner using the same four formulas, using the values x_1, y_1 in the place of x_0, y_0 respectively.

$$f_1(x, y, z) = z$$

$$f_2(x, y, z) = -xz - y$$

By Runge- Kutta method

$k_1 = hf_1(x_0, y_0, z_0)$

$= (0.1)f_1(0,1,0)$

$= (0.1)(0)$

$k_1 = 0$

$l_1 = hf_2(x_0, y_0, z_0)$

$= (0.1)f_2(0,1,0)$

$= (0.1)(0 - 1)$

$l_1 = -0.1$

MILNE'S PREDICTOR AND CORRECTOR METHODS

9. State Milne's predictor-corrector formula.

Solution:

Milne's Predictor Formula:

$$y_{n+1,\ p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_{n})$$

Milne's Corrector Formula:

$$y_{n+1,\ c} = y_{n-1} + \frac{h}{3}(2y'_{n-1} - 4y'_{n} + y'_{n+1})$$

10. Distinguish between single step methods and multi-step methods.

Solution:

single step method	multi-step method
Taylor's series, Euler's, Modified Euler's, Runge – Kutta method of fourth order	Milne's and Adams predictor - corrector method
One prior value is required for finding the value of y at x_i	Four prior value are required for finding the value of y at x_i

11. What are multi-step methods? How are they better than single step methods?

Solution:

One step method: We use data of just one proceeding step.

Multi step method: We use data from more than one of the proceeding steps.

PART-B

TAYLOR SERIES METHOD

1. **Find the value of y at $x = 0.1, 0.2$ given that $\frac{dy}{dx} = x^2y - 1, y(0) = 1$,by Tailor's series method up to four terms.**

Solution:

Given $y' = x^2y - 1$ and $x_0 = 0, y_0 = 1$

We know that Taylor series formula is

$$\boldsymbol{y = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots} \qquad \textbf{... (1)}$$

$y' = x^2y - 1$	$y_0' = 0 - 1 = -1$
$y'' = 2xy + x^2y'$	$y_0'' = 2(0)(1) + 0(-1) = 0$
$y''' = 2[xy' + y] + x^2y'' + y'2x$ $= 2y + 4xy' + x^2y''$	$y_0''' = 2(1) + 4(0)(-1) + 0 = 2$
$y^{iv} = 2y' + 4[xy'' + y'] + x^2y''' + y'''2x$ $= 6y' + 6xy'' + x^2y'''$	$y_0^{iv} = 6(-1) + 6(0)(0) + (0)(2)$ $= -6$

Substituting in equation (1) we get

$$y = 1 + (x-0)(-1) + \frac{(x-0)^2}{2}(0) + \frac{(x-0)^3}{6}(2) + \frac{(x-0)^4}{24}(-6)$$

$$y = 1 - x + \frac{x^3}{3} - \frac{x^4}{4}$$

To find y (0.1)

$$y(0.1) = 1 - 0.1 + \frac{0.1^3}{3} - \frac{0.1^4}{4}$$

$$y(0.1) = 1 - 0.1 + 0.00033 - 0.000025$$

$$y(0.1) = 0.900305$$

To find y (0.2)

$$y(0.2) = 1 - 0.2 + \frac{0.2^3}{3} - \frac{0.2^4}{4}$$

$$y(0.2) = 1 - 0.2 + 0.0026 + -0.0004$$

$$y(0.2) = 0.8022$$

$$x_0 = 0.1\ , y_0 = 00.0993, h = 0.1$$

$$y_2 = y(0.2) = 0.09933 + (0.1)(0.9801334) + \frac{(0.1)^2}{2}(-0.3946868) + \frac{(0.1)^3}{6}(-3.84159)$$

$$y(0.2) = 0.19467$$

2. **Determine the value of $y(0.4)$ using milnes's method given $y' = xy + y^2$, $y(0) = 1$. Using Taylor series method obtain the values of $y(0.1)$ and y(0.2) and y(0.3).**

Solution :

Given $y' = xy + y^2$ and $x_0 = 0, y_0 = 1,$

By Taylor series formula is

$$\boldsymbol{y = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots} \quad \ldots (1)$$

$y' = \quad xy + y^2$	$y_0' = 1$
$y'' = xy' + y + 2\,y'$	$y_0'' = 1 + 2(1)(1) = 1$
$y''' = xy' + y' + y' + 2yy'' + 2y'y'$ $= xy' + 2y'^2 + 2yy'' + 2y'$	$y_0''' = 2 + 6 + 2 = 10$
$y^{iv} = xy''' + y'' + 2y'' + 2y'y'' + 2y\,y''' + 4\;y'y''$ $= xy''' + 3y'' + 4\;y'y'' + 2y\,y'''$	$y_0^{iv} = 9 + 12 + 20 = 41$

Substituting in equation (1) we get

$$y_1 = y(0.1) = 1 + 0.1(1) + \frac{(0.1)^2}{2}(3) + \frac{(0.1)^3}{6}(10) + \frac{(0.1)^4}{24}(41)$$
$$y(0.1) = 1.11684$$

$$y_2 = y(0.2) = 1 + 0.2(1) + \frac{(0.2)^2}{2}(3) + \frac{(0.2)^3}{6}(10) + \frac{(0.2)^4}{24}(41)$$
$$y(0.2) = 1.276067$$

$$y_3 = y(0.3) = 1 + 0.3(1) + \frac{(0.3)^2}{2}(3) + \frac{(0.3)^3}{6}(10) + \frac{(0.)^4}{24}(41)$$
$$y(0.1) = 1.48384$$

X	0	0.1	0.2	0.3
Y	1	1.11684	1.27607	1.49384

$$\boldsymbol{y_{4,\ p} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']}$$

$$y_{4,\ p} = 1 + \frac{4(0.1)}{3}[2(1.35902) - 1.88357 + 2(2.67974)]$$

$$y_{4,\ p} = 1.82586$$

$$y_4' = (0.4)1.82586 + 1.82586^2 \quad = 4.06411$$

By Mile's corrector formula is

$$\boldsymbol{y_{4,\ c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']}$$

$$= 1.27607 + \frac{0.1}{3}[1.88357 + (2.67974) + 4.06411]$$

$$\boldsymbol{y_{4,\ c} = 1.83096}$$

$$y_4 = 1.83096$$

3. **Using Taylor series method fin y at x=1.1 by solving the equation if $\frac{dy}{dx} = x^2 + y^2, y(1) = 2$. Carryout the computations upto fourth order derivative.**

Solution:

Given initial condition $x_0 = 1, y_0 = 2, h = 0.1$

We know that Taylor series formula is

$$y = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots \quad \ldots (1)$$

$y' = \boldsymbol{x^2 + y^2}$	$y_0' = 1 + 2 = 5$
$y'' = 2x + 2yy'$	$y_0'' = 2(1) + 2(2)(5) = 22$
$y''' = 2 + 2yy'' + 2y'^2$	$y_0''' = 2 + 2(2)(22) + 2(5)^2 = 140$
$y^{iv} = 2\,yy''' + 2y'y'' + 4y'y''$	$y_0^{iv} = 2(2)(140) + 2(5)(22) + 4(5)(22) = 1220$

Substituting in equation (1) we get

$$y_1 = 2 + \frac{(x-x_0)}{1!}(5) + \frac{(x-x_0)^2}{2!}(22) + \frac{(x-x_0)^3}{3!}(140) + \frac{(x-x_0)^4}{4!}(1220) + \cdots$$

$$y_1 = 2 + \frac{(1.1-1)}{1!}(5) + \frac{(1.1-1)^2}{2!}(22) + \frac{(1.1-1)^3}{3!}(140) + \frac{(1.1-1)^4}{4!}(1220) + \cdots$$

$$y(1.1) = 2 + 0.1(5) + \frac{(0.1)^2}{2}(22) + \frac{(0.1)^3}{6}(140) + \frac{(0.1)^4}{24}(1220) + \cdots$$

$$y_1 = 2 + 0.5 + 0.11 + 0.023 + 0.00508 = 2.63808$$

EULER AND MODIFIED EULER METHOD

4. **Apply Modified Euler's method to find $y(0.2)$ and $y(0.4)$ given that $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ by taking h=0.2**

Solution:

Initial conditions are

$$x_0 = 0, y_0 = 1, h = 0.2$$

By Euler algorithm

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y_n + \frac{h}{2}(x_n, y_n')$$

$Let\ n = 0$

$$y_1 = y_0 + hf(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}(x_0, y_0')$$

$$= 1 + (0.2)f(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}(\mathbf{0^2 + 1^2}))$$

$$= 1 + (0.2)f(0.1,1.1)$$

$$= 1 + (0.2)\left[(\mathbf{0.1})^2 + (\mathbf{1.1})^2\right]$$

$$= 1 + (0.2)(1.22)$$

$$= 1.244$$

$$y_1 = 1.244$$

$$\boldsymbol{y_1 = y(0.2) = 1.244}$$

$Let\ n = 1,$

$$x_1 = 0.2, y_1 = 1.244, h = 0.2$$

$$y_2 = y_1 + hf(x_1 + \frac{h}{2}, y_1 + \frac{h}{2}(x_1, y_1')$$

$$= 1.244 + (0.2)f(0.2 + \frac{0.2}{2}, 1.244 + \frac{0.2}{2}((\mathbf{0.2})^2 + (\mathbf{1.244})^2))$$

$$= 1.244 + (0.2)f(0.3,1.4028)$$

$$= 1.244 + (0.2)\left[(\mathbf{0.3})^2 + (\mathbf{1.3684})^2\right]$$

$$y_2 = 1.6556$$

$$\boldsymbol{y_2 = y(0.4) = 1.6556}$$

5. Evaluate y at $x = 0.2$ given $\frac{dy}{dx} = y - x^2 + 1,\ y(0) = 0.5$ using modified Euler's method.

Solution:

$$\frac{dy}{dx} = y - x^2 + 1, \qquad x_0 = 0, \qquad y_0 = 0.5\ , h = 0.2$$

By Euler algorithm

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y_n + \frac{1}{2}h(x_n, y_n')$$

$Let n = 0$

$$y_1 = y_0 + hf(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}h(x_0, y_0'))$$

$$= 0.5 + (0.2)f\left(0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2}(0{,}0.5)\right)$$

$f(x_0, y_0) = y_0 - x_0^2 + 1$, f(0,0.5) $= 0.5 + 0 + 1 = 1.5$

$$= 0.5 + (0.2)f[(0.1, 0.5 + 0.1(1.5)]$$

$$= 0.5 + (0.2)f(0.1{,}0.65)$$

$$f(0.1{,}0.65) = 0.65 + (0.1)^2 + 1 = 0.65 - 0.01 + 1$$

$$= 1.65 - 0.01 = 1.64$$

$$y_1 = 0.5 + (0.2)(1.64)$$

$$0.5 + 0.328 = 0.828$$

$$\boldsymbol{y(0.2) = 0.828}$$

6. **Apply Modified Euler's method to find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$**

Solution:

Initial conditions are

$$x_0 = 0, y_0 = 1, h = 0.1$$

By Euler algorithm

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y_n + \frac{1}{2}h(x_n, y_n'))$$

$Let\ n = 0$

$$y_1 = y_0 + hf(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}h(x_0, y_0'))$$

$$= 1 + (0.1)f(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}(\mathbf{1 + 0}))$$

$$= 1 + (0.1)f(0.05{,}1.05)$$

$$= 1 + (0.1)[(\mathbf{0.05})^2 + (\mathbf{1.05})^2]$$

$$= 1 + (0.1)(1.105)$$

$$= 1.1105$$

$$y_1 = 1.1105$$

$$\boldsymbol{y_1 = y(0.1) = 1.1105}$$

$Let\ n = 1,$

$$x_1 = 0.1, y_1 = 1.1105, h = 0.1$$

$$y_2 = y_1 + hf(x_1 + \frac{h}{2}, y_1 + \frac{1}{2}h(x_1, y_1{}'))$$

$$= 1.1105 + (0.1)f(0.1 + \frac{0.1}{2}, 1.1105 + \frac{0.1}{2}((\mathbf{0.2})^2 + (\mathbf{1.1105})^2))$$

$$= 1.1105 + (0.1)f(0.15,1.27321)$$

$$= 1.1105 + 0.1((\mathbf{0.15})^2 + (\mathbf{1.27321})^2))$$

$$y_2 = 1.2749$$

$$\boldsymbol{y_2 = y(0.2) = 1.2749}$$

RUNGE-KUTTA METHOD FOR SOLVING FIRST AND SECOND ORDER EQUATIONS

7. Use Runge-Kutta method of order 4 to find y(1.1) given $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1$,

Solution:

Given $\frac{dy}{dx} = \boldsymbol{y^2 + xy}$, $x_0 = 1, y_0 = 1$and $h = 0.1$

By Runge-kutta method

$$\boldsymbol{k_1 = hf(x_0, y_0)}$$

$$= (0.1)f(1,1)$$

$$= (0.1)(11 + 1)$$

$$k_1 = 0.2$$

$$\boldsymbol{k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)}$$

$$= (0.1)f\left(1 + \frac{0.1}{2}, 1 + \frac{0.2}{2}\right)$$

$$= (0.1)f(1.05,1.1)$$

$$k_2 = 0.2365$$

$$\boldsymbol{k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)}$$

$$= (0.1)f\left(1 + \frac{0.1}{2}, 1 + \frac{0.2365}{2}\right)$$

$$= (0.1)f(1.05, 1.118)$$

$$k_3 = 0.2423$$

$$\boldsymbol{k_4 = hf(x_0 + h, y_0 + k_3)}$$

$$= (0.1)f(1 + 0.1, 1 + 0.2423)$$

$$= (0.1)f(1.1, 1.12423)$$

$$k_4 = 0.2909$$

$$\Delta \boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{6}}(\boldsymbol{k_1} + \mathbf{2}\boldsymbol{k_2} + \mathbf{2}\boldsymbol{k_3} + \boldsymbol{k_4})$$

$$= \frac{1}{6}(0.2 + 2(0.0.2365) + 2(0.2423) + 0.2909)$$

$$\Delta y = 0.2414$$

$$y_1 = y_0 + \Delta y$$

$$= 1 + 0.2414$$

$$\boldsymbol{y(1.05) = 1.2414}$$

To find y(1.1):

Here $x_1 = 1.05, y_1 = 1.2414$and $h = 0.1$

$$\boldsymbol{k_1 = hf(x_1, y_1)}$$

$$= (0.1)f(1.05, 1.2414)$$

$$= (0.1)(2.84454)$$

$$k_1 = 0.28445$$

$$\boldsymbol{k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)}$$

$$= (0.1)f\left(1.05 + \frac{0.1}{2}, 1.2414 + \frac{0.28445}{2}\right)$$

$$= (0.1)f(1.1, 1.3836)$$

$$k_2 = 0.27133$$

$$\boldsymbol{k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)}$$

$$= (0.1)f\left(1.05 + \frac{0.1}{2}, 1.2414 + \frac{0.27133}{2}\right)$$

$$= (0.1)f(1.1, 1.37706)$$

$$k_3 = 0.34110$$

$$\boldsymbol{k_4 = hf(x_1 + h, y_1 + k_3)}$$

$$= (0.1)f(1.05 + 0.1, 1.2421 + 0.34110)$$

$$= (0.1)f(1.15, 1.5825)$$

$$k_4 = 0.43241$$

$$\Delta \boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{6}}(\boldsymbol{k_1} + \mathbf{2}\boldsymbol{k_2} + \mathbf{2}\boldsymbol{k_3} + \boldsymbol{k_4})$$

$$= \frac{1}{6}(0.2844 + 2(0.27133) + 2(0.34110) + 0.43241)$$

$$\Delta y = 0.3236016$$

$$y_2 = y_1 + \Delta y$$

$$= 1.2414 + 0.3236016$$

$$y(1.1) = 1.565001$$

MILNE'S PREDICTOR AND CORRECTOR METHODS

8. Use Milne's predictor – corrector formula to find $\boldsymbol{y(0.4)}$

Given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}, y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12$ *and* $y(0.3) = 1.21$

Solution:

Given $\frac{dy}{dx} = y' = \frac{1}{2}(1 + x^2)y^2$ *and* $h = 0.1$

$$x_0 = 0, x_1 = 0.1, x_2 = 0.2, x_3 = 0.3, x_4 = 0.4, x_5 = 0.5$$

$$y_0 = 1, y_1 = 1.06, y_2 = 1.12, y_3 = 1.21, y_4 = ?$$

Milene's Predictor formula we have,

$$\boldsymbol{y_{n+1}, p = y_{n-3} + \frac{4h}{3}[2y'_{n-2} - y'_{n-1} + 2y'_n} \dots \dots \dots \dots \dots \dots \dots \dots (1)$$

To get y_4, put $n = 3$ in (1) we get

$$y_4, p = y_0 + \frac{4h}{3}[2y_1' - y'_2 + 2y'_3] \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2)$$

$$y'_1 = \left[\frac{1}{2}(1 + x^2)y^2\right]$$

$$= \frac{1}{2}(1 + {x_1}^2){y_1}^2$$

$$= \frac{1}{2}[1 + (0.1)^2](1.06)^2$$

$$y'_1 = 0.56742\dots \dots \dots \dots \dots \dots \dots \dots \dots \quad .(3)$$

$$y'_2 = \frac{1}{2}(1 + {x_2}^2){y_2}^2$$

$$= \frac{1}{2}[1 + (0.2)^2](1.12)^2$$

$$= \frac{1}{2}(1 + 0.04)(1.2544)$$

$$y'_2 = 0.6529 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (4)$$

$$y'_3 = \frac{1}{2}(1 + {x_3}^2){y_3}^2$$

$$= \frac{1}{2}[1 + (0.3)^2](1.21)^2$$

$$= \frac{1}{2}[1 + 0.09](1.464)$$

$$y'_3 = 0.79793 \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (5)$$

Substituting (3),(4) and (5) in (2) we get,

$$y_{4,p} = 1 + \frac{4(0.1)}{2}[2(0.56742) - 0.65229 + 2(0.79793)]$$

$$= 1 + \frac{0.4}{3}[1.13484 - 0.65229 + 1.56586]$$

$$= 1 + 0.27712$$

$$\boldsymbol{y(0.4) = 1.27712}$$

Milne's corrector formula we have

$$\boldsymbol{y_{n+1\,,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_{n} + y_{n+1}\right)}$$

To get y_4, put $n = 3$ we get

$$y_{4,c} = y_2 + \tfrac{h}{3}(y'_2 + 4y_3 + y'_4) \dots \dots \dots \dots\dots \dots \dots \dots (6)$$

$$y'_4 = \frac{1}{2}(1 + {x_4}^2){y_4}^2$$

$$= \frac{1}{2}[1 + (0.4)^2](1.27712)^2$$

$$= \frac{1}{2}(1 + 0.16)(1.63104)$$

$$= \frac{1}{2}(1.16)(1.63104)$$

$$= 0.94600\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ \ldots\ldots\ldots(7)$$

Substituting (4), (5), (7) in (6) we get,

$$y_{4,c} = 1.12 + \frac{0.1}{3}[0.65229 + 4(0.79793) + 0.94600]$$

$$= 1.12 + \frac{0.1}{3}[4.79001]$$

$$= 1.12 + 0.159667$$

$$\boldsymbol{y(0.4) = 1.27966}$$

9. Using Milne's predictor and corrector formulae , find $y(4.4)$ given

$$\boldsymbol{5xy' + y^2 - 2 = 0, y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143}$$

Solution:

Given $y' = \frac{2-y^2}{5x}, x_0 = 4, x_1 = 4.1, x_2 = 4.2, x_3 = 4.3, x_4 = 4.4$

$$y_0 = 1, y_1 = 1.0049, y_2 = 1.0097, y_3 = 1.0143$$

$$y_1' = \frac{2 - y_1^2}{5x_1} = \frac{2 - (1.0049)^2}{5(4.1)} = 0.0493$$

$$y_2' = \frac{2 - y_2^2}{5x_2} = \frac{2 - (1.0097)^2}{5(4.2)} = 0.0467$$

$$y_3' = \frac{2 - y_3^2}{5x_3} = \frac{2 - (1.0143)^2}{5(4.3)} = 0.0452$$

By Mile's predictor formula is

$$\boldsymbol{y_{4,\ p} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']}$$

$$y_{4,\ p} = 1 + \frac{4(0.1)}{3}[2(0.0493 - 0.0467 + 2(0.0452)]$$

$$y_{4,\ p} = 1.01897$$

$$y_4' = \frac{2 - y_4^2}{5x_4} = \frac{2 - (1.1897)^2}{5(4.4)} = 0.0437$$

By Mile's corrector formula is

$$\boldsymbol{y_{4,\ c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']}$$

$$y_{4,\ c} = 1.0097 + \frac{0.1}{3}[0.0467 + 4(0.0452 + 0.0437]$$

$$y_{4,\ c} = 1.01874$$

$$= 1 + \frac{4(0.1)}{3}[2(1.3552) - 1.8535 + 2(2.6589)]$$

$$y_{4,\ p} = 1.8233$$

$$y_4' = x_4 y_4 + {y_4}^2 = (0.4)(1.8233) + (1.8233)^2 = 4.0537$$

By Mile's corrector formula is

$$y_{4,\ c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$y_{4,\ c} = 1.2774 + \frac{0.1}{3}[1.8535 + 4(2.6589) + 4.0537]$$

$$y_{4,\ c} = 1.8165$$

10. Using Runge-kutta method of fourth order, find y for $\boldsymbol{x = 0.1, 0.2, 0.3}$ given that $\boldsymbol{\frac{dy}{dx} = xy + y^2, y(0) = 1}$ Continue the solution at x=0.4 using Milne's method .

Solution:

Given $\frac{dy}{dx} = xy + y^2, x_0 = 0, y_0 = 1, h = 0.1$

By Runge –kutta method

$$k_1 = hf(x_0, y_0)$$
$$= (0.1)f(0,1)$$
$$= (0.1)(\mathbf{0} + \mathbf{1})$$
$$k_1 = 0.1$$
$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$$
$$= (0.1)f(0.05,1.05)$$
$$= (0.1)\big((0.05)(1.05) + (1.05)^2\big)$$
$$k_2 = 0.1155$$
$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1155}{2}\right)$$
$$= (0.1)f(0.05,1.50775)$$
$$= (0.1)\big((0.05)(1.50775) + (1.50775)^2\big)$$
$$k_3 = 0.1172$$
$$k_4 = hf(x_0 + h, y_0 + k_3)$$
$$= (0.1)f(0 + 0.1,1 + 0.1172)$$

$$= (0.1)f(0.1, 1.4424)$$
$$= (0.1)\left((0.1)(1.4424) + (1.4424)^2\right)$$
$$k_4 = 0.1260$$
$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= \frac{1}{6}(0.1 + 2(0.1155) + 2(0.1172) + 0.1260)$$
$$\Delta y = 0.1152$$
$$y_1 = y_0 + \Delta y$$
$$= 1 + 0.1152$$
$$\boldsymbol{y(0.1) = 1.1152}$$

To find y(0.2):

Here $x_1 = 0.1, y_1 = 1.1152$

$$k_1 = hf(x_1, y_1)$$
$$= (0.1)f(0.1, 1.1152)$$
$$= (0.1)\left((0.1)(1.1152) + (1.1152)^2\right)$$
$$k_1 = 0.1255$$
$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$
$$= (0.1)f\left(0.1 + \frac{0.1}{2}, 1.1152 + \frac{0.1255}{2}\right)$$
$$= (0.1)f(0.05, 1.1780)$$
$$= (0.1)\left((0.05)(1.1780) + (1.1780)^2\right)$$
$$k_2 = 0.1355$$
$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$
$$= (0.1)f\left(0.1 + \frac{0.1}{2}, 1 + \frac{0.1355}{2}\right)$$
$$= (0.2)f(0.05, 1.1355)$$
$$= (0.1)\left((0.05)(1.1355) + (1.1355)^2\right)$$
$$k_3 = 0.1577$$
$$k_4 = hf(x_1 + h, y_1 + k_3)$$
$$= (0.1)f(0.1 + 0.1, 1.1152 + 0.1577)$$
$$= (0.1)f(0.2, 1.2729)$$
$$= (0.1)\left((0.2)(1.2729) + (1.2729)^2\right)$$
$$k_4 = 0.1875$$
$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= \frac{1}{6}(0.1255 + 2(0.1355) + 2(0.1577) + 0.1875)$$
$$\Delta y = 0.1499$$
$$y_2 = y_1 + \Delta y$$
$$= 11152 + 0.1499$$
$$\boldsymbol{y(0.2) = 1.2651}$$

To find y(0.3):

Here $x_2 = 0.2, y_2 = 1.2651$

$$k_1 = hf(x_2, y_2)$$
$$= (0.1)f(0.2,1.2651)$$
$$= (0.1)\left((0.2)(1.2651) + (1.2651)^2\right)$$
$$k_1 = 0.1853$$
$$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right)$$
$$= (0.1)f\left(0.2 + \frac{0.1}{2}, 1.2651 + \frac{0.1}{2}\right)$$
$$= (0.1)f(0.25,1.3578)$$
$$= (0.1)\left((0.25)(1.3578) + (1.3578)^2\right)$$
$$k_2 = 0.2183$$
$$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right)$$
$$= (0.1)f\left(0.2 + \frac{0.1}{2}, 1.2651 + \frac{0.2183}{2}\right)$$
$$= (0.1)f(0.25,1.3742)$$
$$= (0.1)\left((0.25)(1.3742) + (1.3742)^2\right)$$
$$k_3 = 0.2232$$
$$k_4 = hf(x_2 + h, y_2 + k_3)$$
$$= (0.1)f(0.2 + 0.1,1.2651 + 0.2232)$$
$$= (0.1)f(0.3,1.4883)$$
$$= (0.1)\left((0.1)(1.4883) + (1.4883)^2\right)$$
$$k_4 = 0.2662$$
$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= \frac{1}{6}(0.1853 + 2(0.2183) + 2(0.2232) + 0.2662)$$
$$\Delta y = 0.2224$$
$$y_3 = y_2 + \Delta y$$
$$= 1.2651 + 0.2224$$
$$y(0.3) = 1.4875$$
$$x_0 = 0, x_1 = 0.1, x_2 = 0.2, x_3 = 0.3, x_4 = 0.4$$
$$y_0 = 1, y_1 = 1.1152, y_2 = 1.2651, y_3 = 1.4875, y_4 = ?$$
$$y' = xy + y^2$$
$$y'_0 = x_0 y_0 + {y_0}^{\mathbf{2}} = (0)(1) + (1)^{\mathbf{2}} = \mathbf{1}$$
$$y'_1 = x_1 y_1 + {y_1}^{\mathbf{2}} = (0.1)(1.1152) + (1.1152)^{\mathbf{2}} = 1.3552$$
$$y'_2 = x_2 y_2 + {y_2}^{\mathbf{2}} = (0.2)(1.2651) + (1.2651)^{\mathbf{2}} = 1.8535$$
$$y'_3 = x_3 y_3 + {y_3}^{\mathbf{2}} = (0.3)(1.4875) + (1.4875)^{\mathbf{2}} = 2.6589$$

By Mile's predictor formula is

$$\boldsymbol{y_{4,\ p} = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]}$$
$$y_{4,\ p} = 1 + \frac{4(0.1)}{3}[2(1.3552) - 1.8535 + 2(2.6589)]$$
$$y_{4,\ p} = 1.8233$$

$$y'_4 = x_4 y_4 + {y_4}^{\mathbf{2}} = (0.4)(1.8233) + (1.8233)^{\mathbf{2}} = 4.0537$$

By Mile's corrector formula is

$$\boldsymbol{y_{4,\ c} = y_2 + \frac{h}{3}[y'_2 + 4y'_3 + y'_4]}$$

$$y_{4,\ c} = 1.2651 + \frac{0.1}{3}[1.8535 + 4(2.6589) + 4.0537]$$

$$y_{4,\ c} = 1.8165$$